

MARKOWITZ THEORY IN PORTFOLIO OPTIMIZATION FOR HEAVY TAILED ASSETS

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Abstract: This paper aims at solving an optimization problem in the presence of heavy tail behavior of financial assets. The question of minimizing risk subjected to a certain expected return or maximizing return for a given expected risk are two objective functions to be solved using Markowitz model. The Markowitz based strategies namely the mean variance portfolio, minimum variance portfolio and equally weighted portfolio are proposed in conjunction with mean and variance analysis of the portfolio. The historical prices of stocks traded at Bursa Malaysia are used for empirical analysis. We employed CAPM in order to investigate the performance of the Markowitz model which was benchmarked with risk adjusted KLSE Composite Index. We performed a backtesting study of portfolio optimization techniques defined under modern portfolio theory in order to find the optimal portfolio. Our findings show that the mean variance portfolio outperformed the other two strategies in terms of performance of investment for heavy tailed assets.

Keywords: portfolio optimization; heavy-tailed; risk and returns; Markowitz model; backtesting

Introduction

Since 1960s, normality assumption on returns of financial asset has been used in many financial models even in a work by Black and Scholes (1973). Such assumption provides a basis for explaining the evolution of asset prices and measuring the market risk. However, highly peaked nature of particular asset prices was recognized by Mandelbrot (1963) which provides an indication of non-normality behavior of the prices. In addition, a stylized fact of heavy tailed is observed in many empirical studies for instance in the paper by Marinelli et al. (2001) and Weron (2008). Recently, several papers such as Oden et al. (2017), Neykov et al. (2014), Dong et al. (2015), Benth and Taib (2013) and Benth et al. (2015) are discussing this critical issue in view of financial assets and commodities.

The heavy tailed asset refers to the financial asset in which returns often possess distribution with tails heavier than those of the normal distribution (see Mandelbrot (1963)). It has implication on the risk management of the financial institution since the acceptance of normally distributed assumption on asset returns may lead to a poor decision, for instance in calculating Value-at-Risk (VaR) (see for example a study by Benth et al. (2015)). We may observe different VaR values if calculated for a heavy-tailed distribution compared to the normal especially for high quantiles of the distribution.

This paper suggested portfolio optimization strategy based on Markowitz (1952) model which aims to allocate the assets genuinely for portfolio diversification. According to Amu and Millegard (2009), the chosen assets and risk factors are important in portfolio theory

since there is no guarantee for profit in the model. Markowitz model determines the efficient set of the portfolio through return, standard deviation (or variance) and coefficient of correlation. Since its introduction, the approaches of mean variance, minimum variance and equally-weighted became the industry standard for asset allocation.

In the mean variance portfolio, the impact of parameter uncertainty on the portfolio selection is one of the main directions of related research. It has been recognized by Frankfurter et al. (1971) and Jobson and Korkie (1980), who revealed that the estimation error obstructed the practical application of portfolio analysis especially in expected returns. This supports Michaud (1989) claims of mean variance portfolio as a “non-financial optimizer” regardless of the benefits of mean variance optimization method. Such optimization is likely to maximize the effect of estimation errors because it estimates the high weight of assets with high expected return, small variance and negative correlation. According to Chopra and Ziemba (1993), the high sensitivity and instability of the estimated mean variance portfolio to estimation errors in the expected returns and covariance may lead to non-robust results and consequently poor performance in terms of their out-of-sample mean and variance. In a paper by DeMiguel and Nogales (2009), their numerical results illustrate the dangerous circumstances of using estimates of mean returns for portfolio selection.

Jorion (1986) and Jagannathan and Ma (2003) have proposed the minimum variance portfolio which relies only on estimation of covariance matrix and not so sensitive to the estimation error. Several studies on minimum variance portfolio are concentrated on highly developed markets (see Bork and Jonsson, 2011). A paper by Haugen and Baker (1991) has constructed a low volatility and long-only portfolio with sensible portfolio constraints when short selling is restricted. It is found that the portfolio with lower volatility outperformed the market portfolio in terms of higher return

and lower volatility (this phenomenon is known as volatility anomaly). An extension work of Haugen and Baker (1991) by Clarke et al. (2006) include a larger time frame and also the econometric techniques to estimating covariance matrix. Their findings support the argument of low volatility anomaly. Chopra and Ziemba (1993) found that estimation error is significantly less than the generated when estimating means. Hence, estimation risk related to minimum variance portfolio is comparatively lesser than mean variance portfolio. This implies that there is no estimation error contributed from the sample mean because minimum variance portfolio depends only on the covariance matrix as an input.

The drawback of portfolio concentration in minimum variance portfolio may be resolved using equally-weighted portfolio which incorporates the same weight to attribute for all assets in the portfolio. A weight of $\frac{1}{N}$ is allocated to each asset at each rebalancing date for N available assets. The approach of equally-weighted portfolio works better than minimum variance portfolio in terms of Sharpe ratio (see DeMiguel et al. (2009a, 2009b)). Therefore, it would serve as a natural benchmark because of its simplicity and low implementation costs. On the other hand, an approach of CAPM which is discovered by Sharpe (1964) and Lintner (1965) is used by investors to estimate the expected return or the performance of stocks in a portfolio. The CAPM employs prediction of risk and the relation between expected return and risk (see Treynor (1999)).

Methods

For any distribution F on \mathbb{R} , the tail function \bar{F} is expressed as

$$\bar{F}(x) = F(x, \infty), \quad x \in \mathbb{R}$$

where $F(x) = \mathbb{P}(X \leq x)$ and $\bar{F}(x) = 1 - F(x)$. For all $\epsilon > 0$, a heavy tailed distribution is given by (see Foss et al. (2013))

$$\int e^{\epsilon x} F(dx) = \infty,$$

or equivalently

$$\frac{\mathbb{P}(X > x)}{e^{-\epsilon x}} \rightarrow \infty.$$

A heavy tail that is possessed by the Pareto distribution is given by (see Pareto (1897))

$$\rho_\alpha(x, \infty) = x^{-\alpha}, \quad x \geq 1$$

where X has a Pareto tail with index $\alpha > 0$. Generally, X has a heavy tailed distribution F if

$$\mathbb{P}(X > x) = x^{-\alpha} L(x)$$

where L is slowly varying,

$$\lim_{t \rightarrow \infty} \frac{L(tx)}{L(t)} = 1, \quad x > 0.$$

Thus, tails of stable distribution of regularly varying with index $-\alpha$ is denoted by

$$1 - F(x) \sim x^{-\alpha}, \quad x \rightarrow \infty.$$

Mean Variance Portfolio

Assume there are N risky assets. At any particular time $t \in \mathbb{R}^+$, the rate of return $R_i(t)$ for the stock price S_i , ($i = 1, \dots, N$) is calculated as

$$R_i(t) = \frac{S_i(t) - S_i(t-1)}{S_i(t-1)},$$

satisfying $R_i \sim \mathcal{N}(\vec{\mu}, \vec{\Sigma})$ where $\vec{\mu} = (\rho_1 \ \rho_2 \ \dots \ \rho_N)$ and $\vec{\Sigma}$ is the covariance matrix. It is also convenient to represent $R_i(t)$ in terms of geometric rate of return as

$$R_i(t) = \ln S(t) - \ln S(t-1). \quad (1)$$

We denote ω_i as the weight for asset i , $\omega = (\omega_1 \ \dots \ \omega_N)^T$ and $\sum_{i=1}^N \omega_i = 1$. The optimal portfolio weights are obtained where the

portfolio achieves an acceptable baseline expected rate of return with minimal volatility in the context of Markowitz theory.

The rate of return of the portfolio, R_p is given as

$$R_p = \sum_{i=1}^N \omega_i R_i.$$

There are two moments of portfolio's rate of return R_p , which are portfolio mean μ_p and portfolio variance σ_p^2 being considered in the mean variance analysis. Assume that the mean μ_i and volatility σ_i of asset i , and the covariance of asset i and j , σ_{ij} are known in the mean variance analysis. Then, portfolio mean is expressed as

$$\begin{aligned} \mu_p &= E(R_p) \\ &= \sum_{i=1}^N E(\omega_i R_i) \\ &= \sum_{i=1}^N \omega_i \mu_i \end{aligned}$$

where $\mu_i = \rho_i$ and the portfolio variance is

$$\begin{aligned} \sigma_p^2 &= Var(R_p) \\ &= \sum_{i=1}^N \sum_{j=1}^N \omega_i \omega_j cov(R_i, R_j) \\ &= \sum_{i=1}^N \sum_{j=1}^N \omega_i \omega_j \sigma_{ij}. \end{aligned}$$

Using notation $\vec{\Sigma}$ and $\vec{\omega}$ for the respective covariance matrix and vector of portfolio's weight, the portfolio variance can be simply represented as

$$\sigma_p^2 = \vec{\omega}^T \vec{\Sigma} \vec{\omega}.$$

The mathematical formulation of mean variance optimization (see Markowitz (1952)) is

$$\begin{aligned} \min_{\omega} \quad & \frac{1}{2} \sum_{i=1}^N \sum_{j=1}^N \omega_i \omega_j \sigma_{ij} \\ \text{subject to} \quad & \sum_{i=1}^N \omega_i \rho_i = \mu_p \\ & \sum_{i=1}^N \omega_i = 1 \end{aligned}$$

where ρ_i is the reward that will depend on the risk aversion of the investor or on the rate of return. In mean variance optimization, two constraints are needed in order to minimize the risk, σ_p^2 for any given expected return. Such optimization problem is solved using Lagrange multiplier method, represented as

$$\mathcal{L} = \frac{1}{2} \sum_{i=1}^N \sum_{j=1}^N \omega_i \omega_j \sigma_{ij} - \lambda_1 \left(\sum_{i=1}^N \omega_i - 1 \right) - \lambda_2 \left(\sum_{i=1}^N \omega_i \rho_i - \mu_p \right), \quad (2)$$

where λ_1 and λ_2 are the budget multiplier and the reward multiplier respectively (usually known as Lagrangian multipliers). The \mathcal{L} is differentiated with respect to ω_i , λ_1 and λ_2 . By setting derivative equal to zero, we have

$$\frac{\partial \mathcal{L}}{\partial \omega_i} = \sum_{j=1}^N \sigma_{ij} \omega_j - \lambda_1 - \lambda_2 \rho_i = 0, \quad (3)$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_1} = \sum_{i=1}^N \omega_i - 1 = 0 \quad (4)$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_2} = \sum_{i=1}^N \omega_i \rho_i - \mu_p = 0. \quad (5)$$

From Eq. 3, the portfolio weight vector is given by

$$\vec{\omega}_p = \vec{\Sigma}^{-1} (\lambda_1 \vec{1} + \lambda_2 \vec{\mu}),$$

where $\vec{1} = (1 \ 1 \dots \ 1)^T$. By applying the following two constraints,

$$\begin{aligned} 1 &= \vec{1}^T \vec{\Sigma}^{-1} \vec{\Sigma} \vec{\omega}_p = \lambda_1 \vec{1}^T \vec{\Sigma}^{-1} \vec{1} + \lambda_2 \vec{1}^T \vec{\Sigma}^{-1} \vec{\mu} \\ \mu_p &= \vec{\mu}^T \vec{\Sigma}^{-1} \vec{\Sigma} \vec{\omega}_p = \lambda_1 \vec{\mu}^T \vec{\Sigma}^{-1} \vec{1} + \lambda_2 \vec{\mu}^T \vec{\Sigma}^{-1} \vec{\mu} \end{aligned}$$

the values λ_1 and λ_2 can be determined. Let $\alpha = \vec{1}^T \vec{\Sigma}^{-1} \vec{1}$, $\beta = \vec{1}^T \vec{\Sigma}^{-1} \vec{\mu}$, $\gamma = \vec{\mu}^T \vec{\Sigma}^{-1} \vec{\mu}$, then we obtain

$$1 = \lambda_1 \alpha + \lambda_2 \beta \quad (6)$$

$$\mu_p = \lambda_1 \beta + \lambda_2 \gamma. \quad (7)$$

Then, by solving λ_1 and λ_2 simultaneously from the Eq. 6 and Eq. 7, we obtain

$$\begin{aligned} \lambda_1 &= \frac{\gamma - \beta \mu_p}{\alpha \gamma - \beta^2} \\ \lambda_2 &= \frac{\alpha \mu_p - \beta}{\alpha \gamma - \beta^2}. \end{aligned}$$

Since $\vec{\Sigma}$ is positive definite, thus $\alpha, \gamma > 0$. By Cauchy-Schwarz inequality, $\alpha \gamma - \beta^2 > 0$. The portfolio variance σ_p^2 for a given value of μ_p is given by

$$\begin{aligned} \sigma_p^2 &= \vec{\omega}_p^T \vec{\Sigma} \vec{\omega}_p \\ &= \vec{\omega}_p^T \vec{\Sigma} (\lambda_1 \vec{\Sigma}^{-1} \vec{1} + \lambda_2 \vec{\Sigma}^{-1} \vec{\mu}) \\ &= \lambda_1 + \lambda_2 \mu_p \\ &= \frac{\alpha \mu_p^2 - 2\beta \mu_p + \gamma}{\alpha \gamma - \beta^2}. \end{aligned}$$

Minimum Variance Portfolio

The portfolio's expected return μ_p is the weighted average of individual asset returns given by

$$\mu_p = \sum_{i=1}^N \mu_i \omega_i = \vec{\mu}^T \vec{\omega}.$$

The positive semi-definite covariance matrix of asset returns denoted by $\vec{\Sigma}$ is represented as

$$\vec{\Sigma} = \begin{bmatrix} \sigma_{1,1} & \cdots & \sigma_{1,N} \\ \vdots & \ddots & \vdots \\ \sigma_{N,1} & \cdots & \sigma_{N,N} \end{bmatrix},$$

where $\sigma_{i,i}$ is the variance of the return of asset i , and $\sigma_{i,j}$ is the variance of return of asset i and j . Thus, the portfolio return's variance, σ_p^2 is

$$\begin{aligned} \sigma_p^2 &= \sum_{i=1}^N \sum_{j=1}^N \omega_i \omega_j \sigma_i \sigma_j \rho_{i,j} \\ &= \vec{\omega}^T \vec{\Sigma} \vec{\omega}. \end{aligned} \quad (8)$$

The Eq. 8 is used as the objective function. Two constraints are needed in order to minimize the risk for any given expected return with consideration for positive and negative weights. A negative weight corresponds to a short-selling position, which is not allowed in certain investment

environment. Therefore, the first constraint is no short-selling position. The second constraint is full investment to ensure that all money available for the investment is completely allocated. Mathematically, the constraints are defined as

Long-only investment: $\bar{\omega}_i \geq 0, \forall i$

Full investment:

$$\sum_{i=1}^N \omega_i = \bar{\omega}^T \bar{1} = 1, \quad \bar{1} = (1 \ 1 \dots \ 1)^T.$$

Hence, the minimum variance portfolio is found by solving the quadratic equation with risky assets to produce the portfolio with lowest risk as

$$\begin{aligned} \min_{\omega} \quad & \bar{\omega}^T \bar{\Sigma} \bar{\omega} \\ \text{subject to} \quad & \bar{\omega}^T \bar{1} = 1 \\ & \bar{\omega}_i \geq 0. \end{aligned}$$

Such optimization problem is solved by Lagrangian method. Using Lagrangian multiplier λ , the minimum variance portfolio is solved as

$$\mathcal{L}(\bar{\omega}, \lambda) = \bar{\omega}^T \bar{\Sigma} \bar{\omega} - \lambda(\bar{\omega}^T \bar{1} - 1). \quad (9)$$

Taking the partial derivatives for ω and λ of Eq. 9 and by equating to zero,

$$\frac{\partial \mathcal{L}(\bar{\omega}, \lambda)}{\partial \bar{\omega}} : \bar{\Sigma} \bar{\omega} - \lambda \bar{1} = 0, \quad (10)$$

$$\frac{\partial \mathcal{L}(\bar{\omega}, \lambda)}{\partial \lambda} : \bar{\omega}^T \bar{1} - 1 = 0. \quad (11)$$

The above system is solved by substitution. From Eq. 10, $\bar{\omega} = \lambda \bar{\Sigma}^{-1} \bar{1}$. The respective $\bar{\omega}$ is substituted into Eq. 11 to obtain $\lambda = \frac{1}{(\bar{1}^T \bar{\Sigma}^{-1} \bar{1})}$. The final minimum variance portfolio weight vector is given by

$$\bar{\omega}_p = \frac{\bar{\Sigma}^{-1} \bar{1}}{(\bar{1}^T \bar{\Sigma}^{-1} \bar{1})}$$

Equally-weighted Portfolio

The use of equally-weighted portfolio is simpler compared to the other two

aforementioned strategies due to its independency from historical returns. In this paper, the index is constructed by allocating a “ $\frac{1}{2}$ ” portion of the portfolio to each of the two assets when the weighted index is rebalanced. The expected return for equally weighted portfolio is the sum of the expected returns to the assets divided by the number of assets. The variance of equally-weighted portfolio is denoted as

$$\sigma_p^2 = \frac{1}{N} \bar{\sigma}^2 + \frac{N-1}{N} \bar{\Sigma},$$

where N is the number of assets in the portfolio, $\bar{\sigma}^2$ is the average variance of the assets, and $\bar{\Sigma}$ is the average covariance of the assets.

Capital Asset Pricing Model (CAPM)

The central insight of CAPM is that the riskiness of an asset is measured by its market beta but not by the standard deviation in equilibrium. Let R_m, R_p and R_s be the market’s simple return, portfolio’s simple return and stock’s simple return respectively. There is a linear relationship between expected return of portfolio and expected return of market, given by

$$\mathbb{E}(R_s) = r_f + \beta [\mathbb{E}(R_m) - r_f]$$

where \mathbb{E} denotes an expectation, r_f is the risk free rate and β is the stock’s beta expressed as

$$\beta := \frac{\text{cov}(R_s, R_m)}{\text{Var}(R_m)}.$$

Performance Evaluation

A number of performance metrics are used to measure the attractiveness of implementing the mean variance portfolio, min variance portfolio, and equally-weighted portfolio by measuring the returns compared to the benchmark risk-adjusted basis. Sharpe ratio and Treynor ratio are used to measure the risk-return tradeoff. The Sharpe ratio is computed as the difference between mean return with the risk-free rate divided by the total risk of the portfolio, or simply (see Sharpe (1964))

$$\text{Sharpe ratio} = \frac{R_p - r_f}{\sigma_p}$$

The higher value of Sharpe ratio, the more attractive is the risk-adjusted return. Active premium is a measure of the annualized return of the benchmark subtracted by the investment annualized return to yield the investment's gain or loss that is over or under the benchmark. The positive active premium is good whereas the negative active premium is poor.

Treynor ratio (also known as reward-to-volatility ratio) is a risk-adjusted measurement of return and is calculated as average return of a portfolio, R_p minus the average return of risk-free rate, r_f divided by beta of the portfolio, β_p , or simply (see Treynor (1999))

$$\text{Treynor ratio} = \frac{R_p - r_f}{\beta_p}$$

The maximum drawdown, MDD is also used to calculate the maximum percentage loss of the portfolio value. In mathematical notation, MDD is expressed as

$$\text{MDD} = \min_{\forall t, T} \left(\prod_t^T r_{i,t} - 1 \right)$$

The maximum drawdown is important to investor in identifying the worst possible scenarios which allow investor to identify the required recovery rate to balance with their previous high recorded value. Moreover, Jensen's alpha is constructed in CAPM which is used to measure the performance of an investment strategy. It is the one factor market model that the excess return of an investment relative to the return of a benchmark and often used in conjunction with beta. Given parameters μ_{RD} as mean return of dependent variable and μ_R as mean return of independent variable, the values of alpha a and annualized alpha A are calculated as

$$a = \mu_{RD} - \beta \times \mu_R$$

$$A \text{ (monthly data)} = ((1 + a)^{12}) - 1.$$

The information ratio, IR, is more or less similar to the Sharpe ratio where both are used to evaluate the risk-adjusted rate of return of a portfolio. The information ratio is computed as return of the portfolio less the return of its benchmark with its tracking error,

$$\text{IR} = \frac{R_p - R_i}{S_{p-i}}$$

where R_i is the return of its benchmark and S_{p-i} is the tracking error. The tracking error is the standard deviation of the difference between return of the portfolio and the return of its benchmark. In order to get a high information ratio, the tracking error must be small, so that the volatility is low and thus the information ratio is high. The higher the information ratio, the better the consistency.

Results and Discussion

Empirical Analysis of Stock Prices

In Figure 1, we plotted the time series of stock prices of Maybank together with Public Bank for 10 years starting from January 1, 2007 until December 31, 2016. Both time series show the increasing trend at different rates. These prices had a cascade effect during the period of 400 to 600 days as a reflection of Global Financial Crisis during the financial year 2008-2009. The price of Maybank and Public Bank declined 26.6% and 19.55% compared to 12.2% and 41.94% respective gain in the previous financial year. It is also obvious that Maybank's prices were highly volatile compared to the prices of Public Bank.

Next, we computed the daily logarithmic returns of both assets using Eq. (1) as illustrated in Figure 2. We observed a volatility clustering property in both time series in which the amplitudes of price changed accordingly. In portfolio performance comparison, cumulative daily logarithmic returns were used as illustrated in Figure 3.

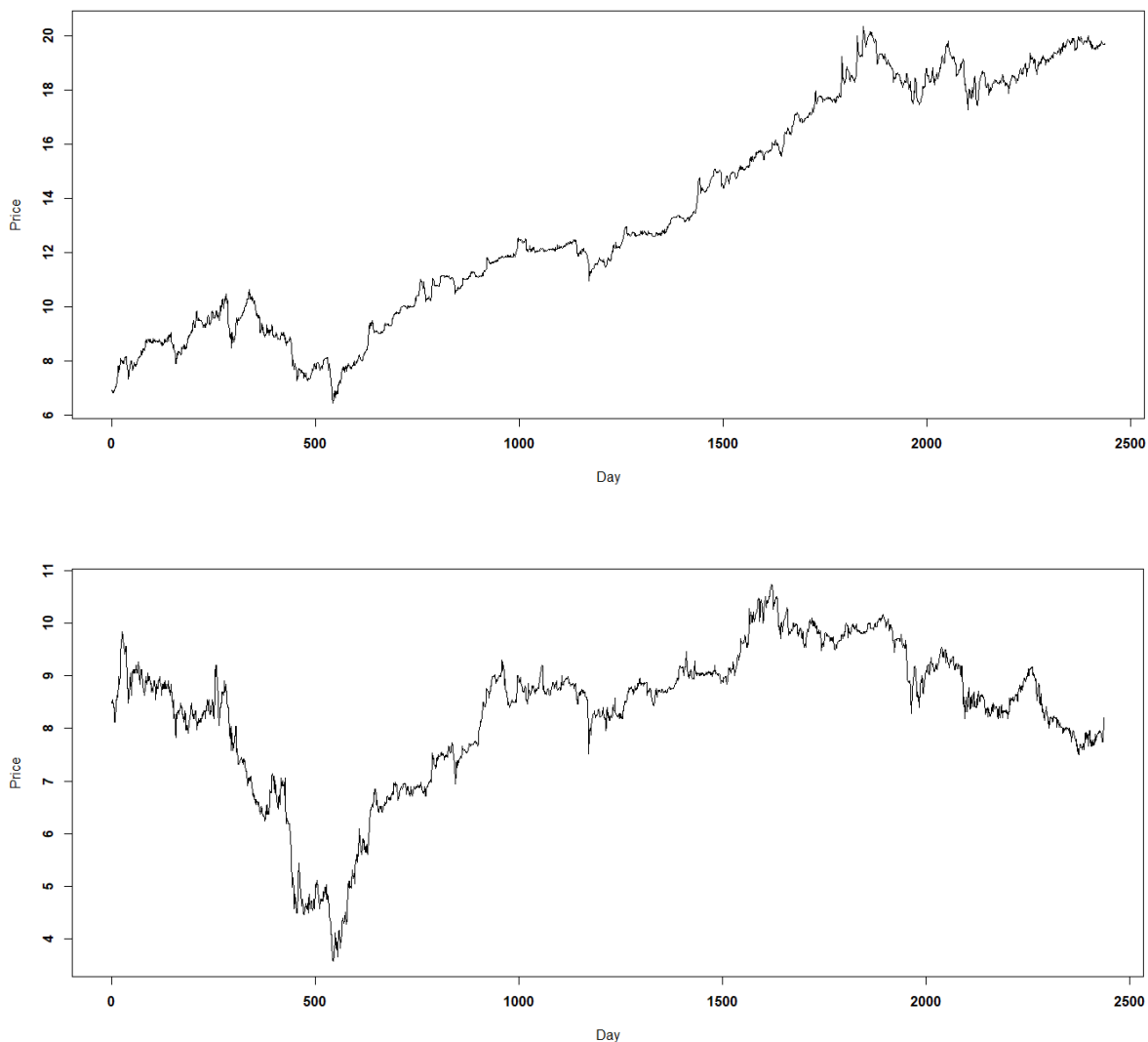


Figure 1: Time series of Maybank (top) and Public Bank (bottom) stock prices.

The empirical distributions of logreturns are shown in Figure 4 together with fitted normal distribution. In Figure 5, we show the QQ-plot for the logarithmic returns of Maybank and Public Bank. The fitted normal distribution and QQ-plots clearly show that the tails of both returns distribution were heavier than the tails of normal distribution. These indicate that the increment of both asset prices had heavy-tailed behavior and deviated much from normality.

Portfolio Optimization

Table 1 (top) reports the optimal weights, ω which were 0.226 and 0.774 and computed by minimum variance optimization for Maybank

and Public Bank respectively. In Table 1 (bottom) the reported optimal weights computed using mean variance optimization for Maybank and Public Bank were 0.204000 and 0.796000 respectively.

The risks to return of two portfolio combinations are plotted in Figure 6. The tradeoff between risk and return is the high risk is associated with a larger probability of high return, while low risk is associated to larger potential of low return.

Backtest and Compute Return

The backtesting of the portfolios throughout this research was accompanied with rebalancing strategy on a yearly basis or 250

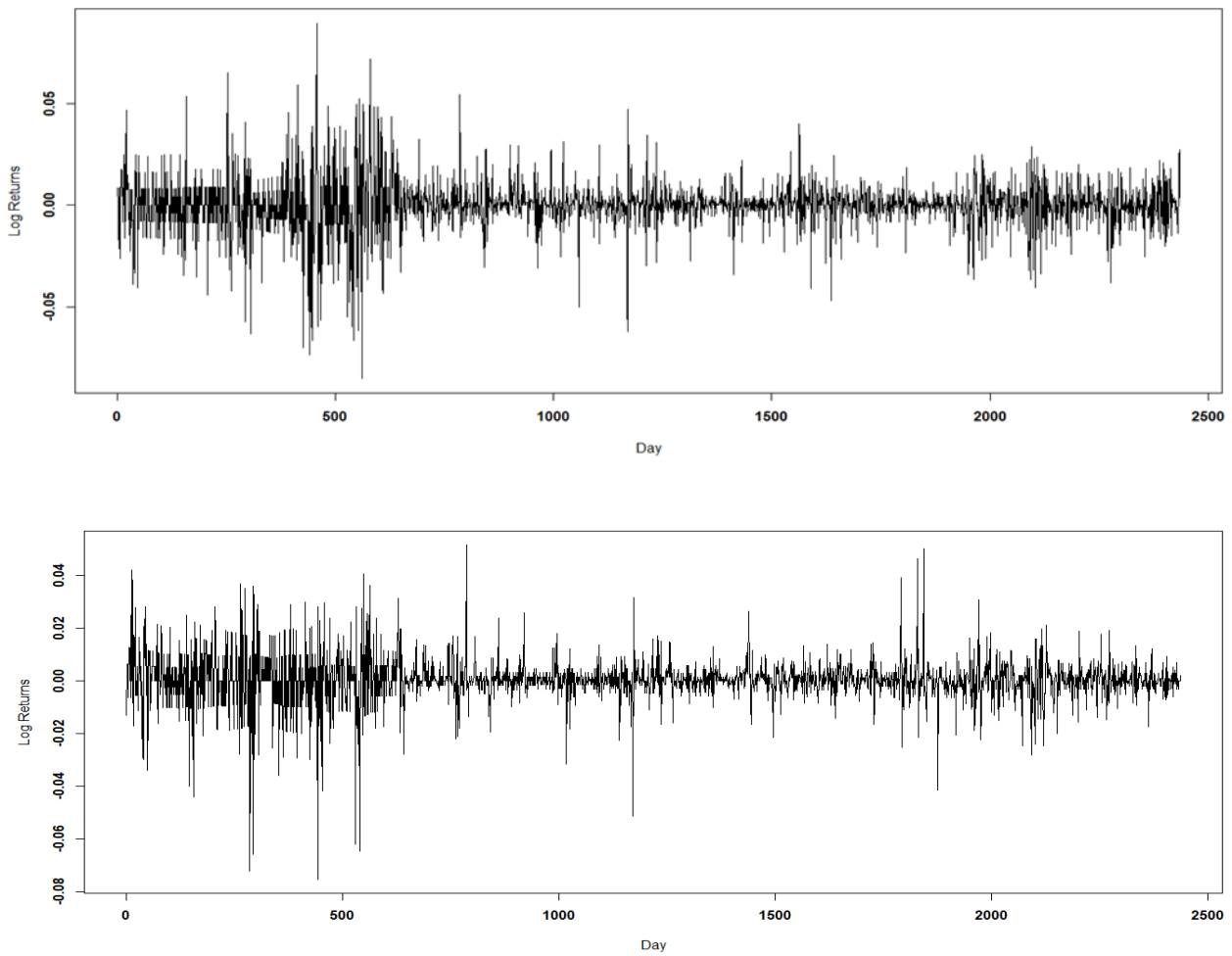


Figure 2: Time series of logarithmic returns of Maybank (top) and Public Bank (bottom).

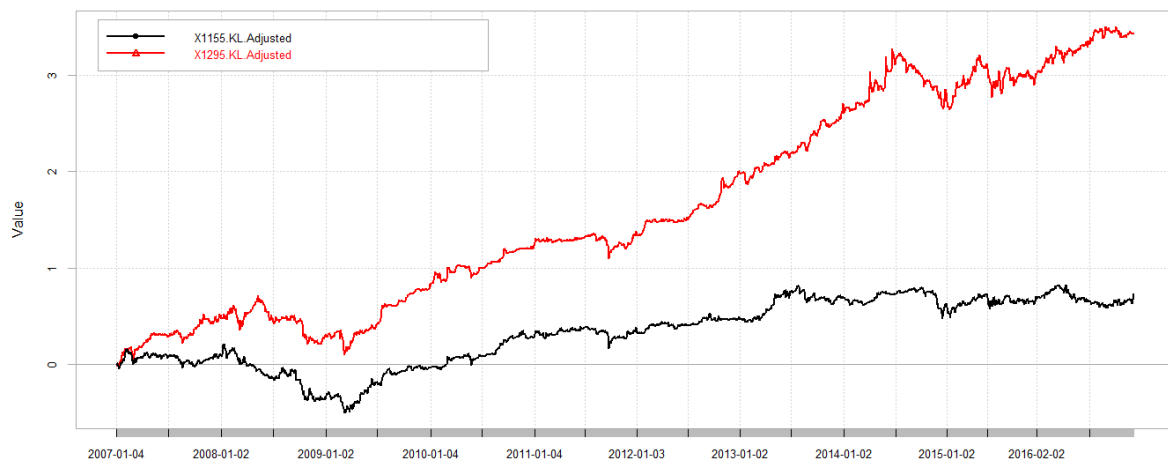


Figure 3: The cumulative daily logarithmic returns of Maybank and Public Bank

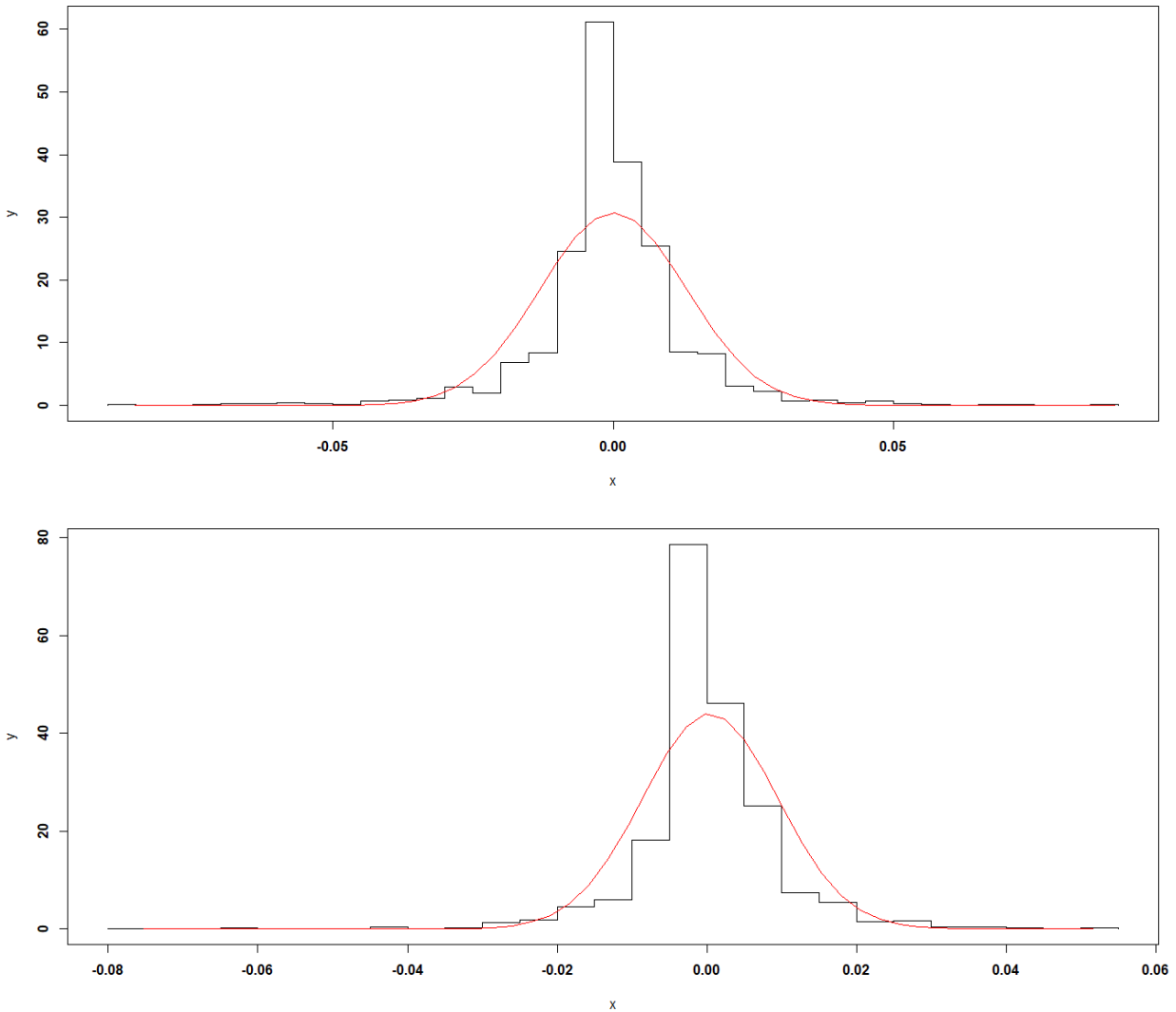


Figure 4: Density plot of logreturns of Maybank (top) and Public Bank (bottom) together with density of normal distribution.

trading days of rolling periods. The backtest bought all stocks that passed the strategy on the start date of the backtest which was the date January 1, 2007. When the backtest balanced, the portfolio’s risk and return characteristics could be recaptured where it also gave equal portfolio weights to each stock in the strategy at each rebalance. Thus, rebalancing strategy is to minimize the risk rather than maximize the return in an asset allocation. We found that the optimized minimum variance and mean variance portfolio are 0.2 and 0.8 for Maybank and Public Bank respectively.

Portfolio Performance

Figure 7 shows the cumulative return for the strategies of min variance portfolio, mean variance portfolio and equally-weighted portfolio based on the first 6 years of research window from January 1, 2007 to December 31, 2012 which is benchmarked with the risk Adjusted KLSE index. The Adjusted KLSE index outperformed the three strategies during year 2008 to 2009. However, at the beginning of year 2010, simulated returns outperformed Adjusted KLSE index in which the time series of three strategies lie above the Adjusted KLSE index.

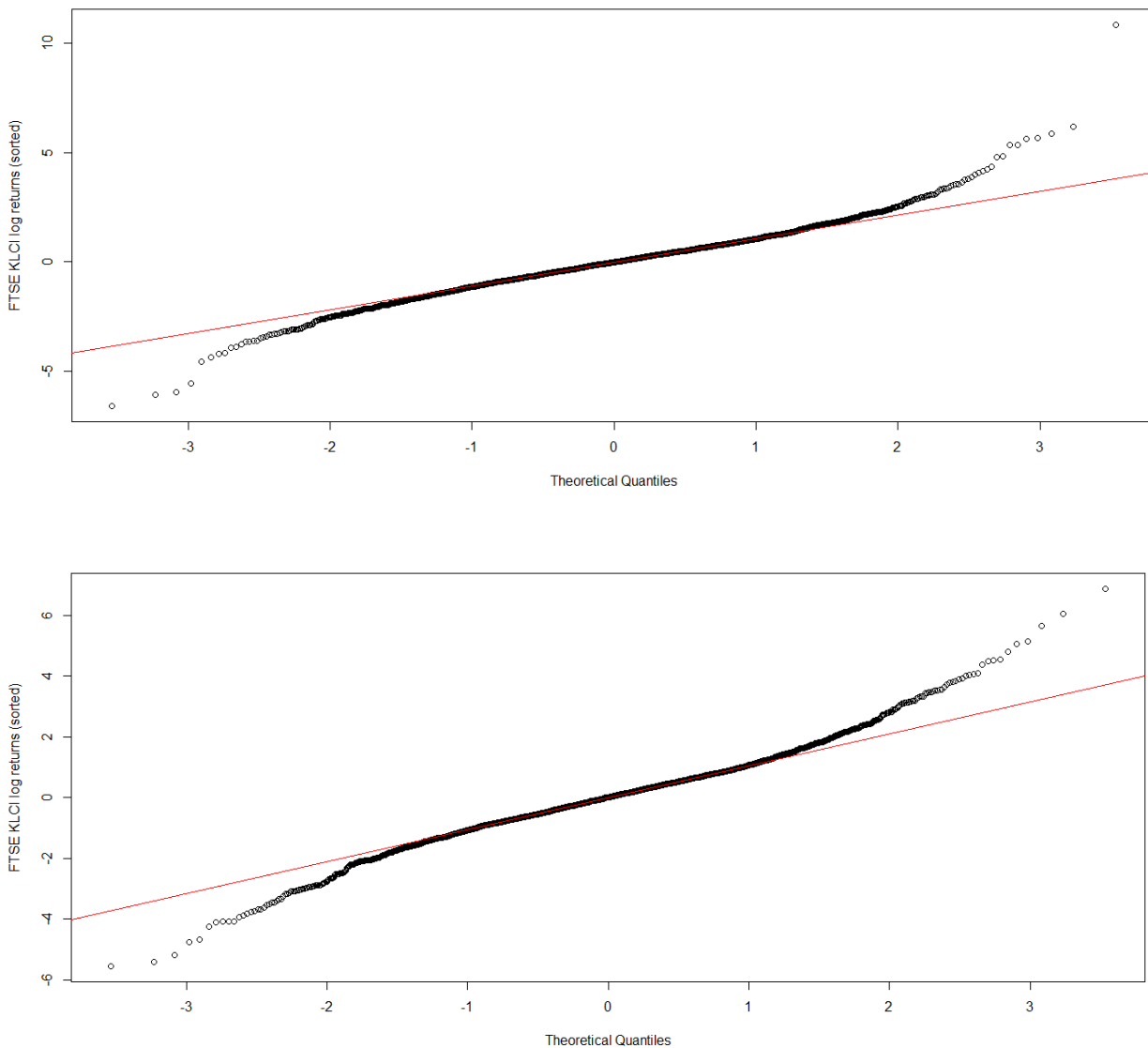


Figure 5: The QQ-plot of the logreturns of Maybank (top) and Public Bank (bottom).

Table 2 reports the summary of backtest statistics for the min variance portfolio, mean variance portfolio, equally-weighted portfolio and Adjusted KLSE. The min variance portfolio has the highest annualized return of 22.73% which outperforms the Adjusted KLSE. In addition, the strategy with highest annualized standard deviation of 0.1551 is equally-weighted portfolio. The min variance portfolio and mean variance portfolio are considered to have a good return since their Sharpe ratio is greater than 1. The mean variance portfolio has the smallest maximum drawdown of 0.2060 and on the other hand Adjusted KLSE has the worst drawdown of

0.4686. These statistics indicate that mean variance portfolio is the most preferred strategy for its small investment losses.

The set of returns generated by optimized min variance portfolio, mean variance portfolio and equally-weighted portfolio relative to risk Adjusted KLSE resulting in a set of measures which is related to excess return single index model or CAPM as shown in Table 3. The betas of the three strategies are lower than 1, which theoretically less volatile than the market. The annualized alpha of mean variance portfolio

Table 1: Optimal weights of minimum variance optimization (top) and mean variance optimization (bottom)

| Type | Value of objective function, x^* | Best value | |
|-----------|------------------------------------|------------|----------|
| Iteration | | x_1 | x_2 |
| 1 | 0.008413 | 0.226000 | 0.774000 |
| 2 | 0.008413 | 0.226000 | 0.774000 |
| 3 | 0.008413 | 0.226000 | 0.774000 |
| 4 | 0.008413 | 0.226000 | 0.774000 |
| 5 | 0.008413 | 0.226000 | 0.774000 |

| Type | Value of objective function, x^* | Best value | |
|-----------|------------------------------------|------------|----------|
| Iteration | | x_1 | x_2 |
| 1 | 0.007805 | 0.204000 | 0.796000 |
| 2 | 0.007805 | 0.204000 | 0.796000 |
| 3 | 0.007805 | 0.204000 | 0.796000 |
| 4 | 0.007805 | 0.204000 | 0.796000 |
| 5 | 0.007805 | 0.204000 | 0.796000 |

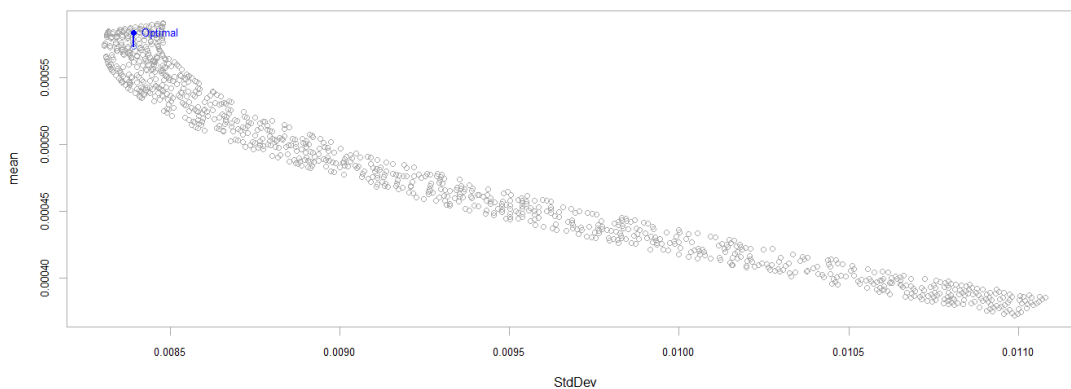


Figure 6: Risk to return

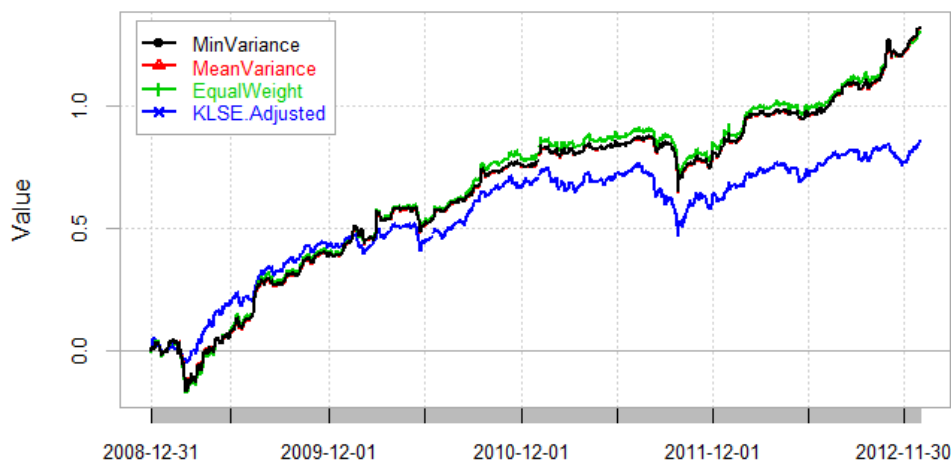


Figure 7: Cumulative returns of min variance portfolio, mean variance portfolio and equally-weighted portfolio together with Adjusted KLSE

Table 2: Backtest statistics for min variance portfolio (MinVP), mean variance portfolio (MeanVP), equally-weighted portfolio (EWP) and Adjusted KLSE (Adj. KLSE)

| | MinVP | MeanVP | EWP | Adj. KLSE |
|---|--------|--------|--------|-----------|
| Annualized return | 0.2273 | 0.2270 | 0.1281 | 0.0580 |
| Annualized standard deviation | 0.1235 | 0.1232 | 0.1551 | 0.1406 |
| Annualized Sharpe ($R_f = 0\%$) | 1.8402 | 1.8422 | 0.8255 | 0.4130 |
| Maximum drawdown (MDD) | 0.2087 | 0.2060 | 0.4452 | 0.4686 |

Table 3: A summary of Capital Asset Pricing Model

| | MinVP to Adj. KLSE | MeanVP to Adj. KLSE | EWP to Adj. KLSE |
|--------------------------|--------------------|---------------------|------------------|
| Beta | 0.7391 | 0.7290 | 0.8064 |
| Annualized Alpha | 0.1126 | 0.1143 | 0.0897 |
| Tracking Error | 0.0996 | 0.1002 | 0.1150 |
| Active Premium | 0.0750 | 0.0749 | 0.0782 |
| Information Ratio | 0.7526 | 0.7477 | 0.6795 |
| Treynor Ratio | 0.3076 | 0.3113 | 0.1588 |

relative to the benchmark is 11.43% which is the highest among the other strategies. Generally, higher alpha means that the portfolio is tracking better than the benchmark index. Since low tracking error means low volatility, min variance portfolio has the lowest volatility while equally-weighted portfolio has the highest volatility. The min variance portfolio in relation to Adjusted KLSE has the lowest tracking error with highest information ratio of 0.7526. Furthermore, the mean variance portfolio relative to Adjusted KLSE has the highest Treynor ratio, therefore having higher rate of return. In order to deliver the portfolio with the lowest volatility, the min variance portfolio favours value stocks and avoid faster-growing, longer duration growth stocks with high consistency because they exhibit higher price volatility.

Conclusion

In this paper, we studied the optimization problem for heavy tailed asset using Markowitz model. Such model is used to minimize risk

subjected to a given expected return or on the contrary to maximize return subjected to a given expected risk. In portfolio theory, it is very important for investors to choose the type of assets to invest and also the risk factors. The model can be solved using Lagrange multipliers. We used CAPM as a tool to evaluate the performance of Markowitz model and estimate the expected return of the portfolio. The analysis has been focused on the investment strategies relative to a benchmark with a yearly basis rebalancing.

We found that the min variance portfolio, mean variance portfolio and equally-weighted portfolio strategies have taken turns in outperforming one another. In backtesting, the strategy that generated the highest annualized return has the highest annualized standard deviation and vice versa. It is concluded that CAPM is reasonable to be the indicator of stock prices in Malaysia from 2007 to 2012. The min variance portfolio has generated greater risk adjusted returns than the benchmark in CAPM model. The strategy exhibits lower volatility,

has generated returns persistently and demonstrated positive alphas throughout the research window. In summary, Markowitz model is found to be useful in portfolio optimization for heavy tailed asset. The finding is parallel with the study done by Mainik et al. (2015). Moreover, investors could use CAPM to estimate the behaviour of the stocks in Malaysia in minimizing the downside risk and invest rationally in stock market. Furthermore, it is suggested to diversify a portfolio to reduce the unsystematic risk and to increase the confidence of investors towards investment decision. However, further study might apply the same method in a wider dataset in order to check for possible biases. A deeper investigation of the Markowitz model for heavy-tailed assets is needed since there are issues on market efficiency and the relevant of normality assumption for returns distribution in the classical Markowitz.

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