

EXCHANGE RATE FORECASTING USING FUZZY TIME SERIES-MARKOV CHAIN

LIM XIN HUI AND BINYAMIN YUSOFF*

Faculty of Ocean Engineering Technology and Informatics, Universiti Malaysia Terengganu, 21030 Kuala Nerus, Terengganu, Malaysia.

*Corresponding author: binyamin@umt.edu.my

Abstract: Exchange rate forecasting plays an important role in financial management. However, it is a complex process with high nonlinearity and data irregularity. Moreover, the forecasting of exchange rate is highly involved with imprecise and uncertain data. Analysis of forecasting models which corresponds to the exchange rate has always experienced fluctuations. Therefore, exchange rate forecasting becomes a challenging task in finance. Several studies have shown that stand-alone forecasting models such as time series, fuzzy time series, and Markov chain have their own drawbacks and are not successful enough in forecasting accurately. In this study, we propose a hybrid model of fuzzy time series-Markov chain to forecast the future exchange rate. Fuzzy time series-Markov chain is a combination of the classic fuzzy time series model with Markov chain model used to analyse a set of time series data. The main motivation for this study is to improve the accuracy in exchange rate forecasting. The selected currencies are Malaysian Ringgit (MYR) and Singapore Dollar (SGD). The proposed model was then evaluated by the Mean Absolute Percentage Error (MAPE) performance metric to test the robustness of the model. Lastly, a comparison between the proposed model and fuzzy time series model was conducted with respect to the MAPE. The results showed that the MAPE value for fuzzy time series-Markov chain was 0.9895% which fell under the criterion of highly accurate forecasting. Meanwhile, the MAPE value for fuzzy time series was 3.4306%. Thus, the forecasting performance of the proposed model was better than the fuzzy time series model. This study reveals the potential benefits of the proposed model as a highly accurate forecasting model.

Keywords: Exchange rate forecasting, fuzzy set theory, time series, fuzzy time series, markov chain, fuzzy time series Markov chain, MAPE

Introduction

Exchange rate is an important consideration in the globalized market. Hence, forecasting of the exchange rate plays a crucial role in international financial management. It provides information on the current states and changes in exchange rate. A successful prediction of future exchange rate can yield a significant profit, assisting company management and investors to make an informed decision in financial decision-making process.

Forecasting involves making an informed prediction to determine future trends. It is the most essential method in financial management. In general, there are two types of forecasting

methods reported in the literature: qualitative and quantitative models. The qualitative model is based on non-statistical approaches such as subjective judgments or expert opinion, and personal experiences while the quantitative model relies on objective and statistical approach in forecasting. Some of the examples of forecasting models are market survey, simple mean, simple moving average, exponential smoothing, Markov chain, linear regression, and multiple regression (Reid & Sanders, 2010).

The time series models are one of the most important elements in forecasting (Islam & Hossain, 2020). Time series forecasting is a method that collects and analyses previous

observations or experiences of the same system to develop a model describing an underlying relationship. It uses time as an independent variable in the relationship. There are two types of time series: discrete and continuous. The measurements of time series models are usually daily, weekly, monthly, quarterly, annually or any suitable time intervals (Chatfield, 2000).

Time series model helps us to analyse past behaviour and compare the predicted result to the past performance. However, the time series model does not always meet a satisfactory result in exchange rate forecasting. The exchange rate is influenced by some other parameters including the psychological and sociological factors that cannot easily be measured and are very noisy. It tends to fluctuate unexpectedly. Besides, time series models are linear and have chaotic behaviours (Erdogan & Goksu, 2014). It is unable to capture the nonlinear relationship in the movement of the exchange rate.

Markov chain, as proposed by Andrey Andreyevich Markov, is another widely used forecasting model in finance (Seneta, 2006). It is a stochastic process with 'no-after effect' property and its state space is discrete. Markov model only considers the characteristic of the historical data and predicts changes by using transition probability. Because of its 'no-after effect' property, it is more effective under the market mechanism (Zhang & Zhang, 2009). Since the Markov chain is a probability forecasting method, it only provides the predicted result in probability state rather than a proper value. Moreover, it also requires the data to be accurate and precise.

Despite the importance of time series and Markov chain models for forecasting, other issues related to these models have been reported in the literature. These include uncertainty problems due to ambiguity or imprecision. The real application of these models is usually accompanied by this type of uncertainty. Hence, the models become unstable and unreliable when uncertainty exists. Zadeh (1965) introduced a fuzzy set theory to cope with this type of uncertainty. The linguistic variable

is an important concept in fuzzy set theory, transforming values into words or sentences in the natural or artificial language (Zadeh, 1973). It collects elements into similar groups so that we can deal with less precise data and handle more complex systems. Fuzzy logic, on the other hand, is based on fuzzy set concept and is a flexible model to deal with human reasoning and decision making. It can add more functionality and allows the imprecise data in the model (Meyer & Zimmermann, 2011).

For the time series forecasting, if the historical data are insufficient or associated with linguistic values, vagueness and imprecision, the applicability of these models will be affected. To cope with this uncertainty in the data, Song and Chissom (1993; 1994) proposed a fuzzy time series model to handle the forecasting problems using fuzzy logic. Later, the simplification of this model was proposed by Chen (1996). This model has shown its effectiveness in the foreign exchange problem (Boiroju *et al.*, 2011). It also has more capability in handling the nonlinear relationships. Although the uncertainty represented by the linguistic terms increases the interpretability of the data, but there are still some limitations to the fuzzy time series model. First, the temporal patterns were defined by rigid regions that were hard to adjust when there is noise exist. Next, it requires complex computational skills and a lack of stability (Tayal, 2011).

Similarly, considering that the uncertainty exists in realistic states, the Markov chain may be less effective to deal with the complex task. Hence the fuzzy version of Markov chain has been proposed in the literature to tackle this problem (Li & Xiu, 2014; Uzun & Kiral, 2017). Avrachenkov and Sanchez (2002) have shown that the fuzzy Markov chain is a robust system with respect to small perturbations of the transition matrix, which is not the case for the classical probabilistic Markov chains.

As stated above, the movement of exchange rate is affected by various factors. Therefore, no single method can take those considerations into account and predict the changes in exchange

rate accurately. Fortunately, we can combine different methods to improve the performance and reduce the forecasted error of the stand-alone models. For example, Tsaur (2012) has proposed a fuzzy time series-Markov chain model to overcome the above highlighted problems. The efficacy of the proposed model has been shown in forecasting the exchange rate between Taiwan and US dollar (Tsaur, 2012). Thus, with the hybrid model, it is implied that the more the information pertaining to the system dynamics is induced, the better the forecasting will be. Therefore, in this study, we propose to apply the fuzzy time series-Markov chain model to forecast the exchange rate between the Malaysian Ringgit (MYR) and Singapore Dollar (SGD) based on monthly time series data.

Various performance measures have been proposed to estimate the forecast accuracy and to test the robustness of the models (Sahin *et al.*, 2014). These are known as performance metrics. The examples of these measures are mean forecast error (MFE), mean percentage error (MPE), mean absolute percentage error (MAPE), mean squared error (MSE), and sum of squared error (SSE), among others (Adhikari & Agrawal, 2013). In this study, MAPE is used to test the robustness of the proposed model. This selection is due to its advantages of scale-independency and interpretability.

The remainder of the paper is organized as follows: Section 2 is devoted to the preliminaries where the definitions and basic concepts of fuzzy set theory, fuzzy time series, and Markov chain are provided. Section 3 introduces the methodology of the proposed model: fuzzy time series-Markov chain model and MAPE performance metric. In Section 4, the case study of forecasting the exchange rate between MYR and SGD is put forward. Furthermore, Section 5 presents the results and discussion of this study. Finally, some concluding remarks are given in Section 6.

Preliminaries

Fuzzy Set Theory

Fuzzy set theory states that a certain element may belong or simultaneously not belong to the same set of certain degree. This theory can be mathematically defined as the following:

Definition 1 (Zadeh, 1965): A fuzzy set in a universe X is a set that is defined as:

$$\tilde{A} = \{(\mu_{\tilde{A}}(x), x) | x \in X\}, \tag{1}$$

where $\mu_{\tilde{A}}(x) \in [0,1]$ represents the degree of membership of element x in \tilde{A} .

Definition 2 (Chang & Zadeh, 1972): A fuzzy number $\tilde{A} = (a_1, a_2, a_3)$ where $a_1 < a_2 < a_3$ is called triangular fuzzy number if its membership function is given by:

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x - a_1}{a_2 - a_1}, & \text{if } a_1 \leq x \leq a_2 \\ \frac{a_3 - x}{a_3 - a_2}, & \text{if } a_2 \leq x \leq a_3 \\ 0, & \text{otherwise.} \end{cases} \tag{2}$$

The other types of membership functions are trapezoidal membership function, Gaussian membership function, sigmoid membership function, to name a few (Madi, *et al.*, 2020; Ali, *et al.*, 2015).

Fuzzy Time Series

In the following, the definitions and basic concept of fuzzy time series are presented. Song and Chissom (1993) first established the definitions of fuzzy time series. Then, the simplification of this model was proposed by Chen (1996). The definitions presented here are merely based on Chen (1996).

Definition 3 (Chen, 1996): Let U be the universal discourse with $U = (u_1, u_2, \dots, u_n)$. A fuzzy set A_i in the universal discourse, U is defined as:

$$A_i = \frac{f_{A_i}(u_1)}{u_1} + \frac{f_{A_i}(u_2)}{u_2} + \dots + \frac{f_{A_i}(u_n)}{u_n}, \tag{3}$$

where f_{A_i} represents the membership function of fuzzy set A_i , u_j is an element in A_i and $f_{A_i}(u_j)$ is the degree of membership of u_j , $f_{A_i}(u_j) \in [0,1]$ and $1 \leq j \leq n$.

Definition 4 (Chen, 1996): Suppose $Y(t-1) = A_i$ and $Y(t) = A_j$ where A_i and A_j are fuzzy set, the fuzzy logical relationship between $Y(t-1)$ and $Y(t)$ can be expressed as $A_i \rightarrow A_j$ where,

$Y(t-1)$: collected data at time $t-1$

$Y(t)$: collected data at time t

A_i : current set of the fuzzy logical relationship

A_j : next state of the fuzzy logical relationship

Markov Chain

Markov Chain is a stochastic process with Markov property. It is a time series model with discrete state (He & Jiang, 2018).

Definition 5 (Adesokan *et al.*, 2017): A Markov chain is a collection of random variables (where the index runs through 0,1, ...) having

the property that, given the present, the future is conditionally independent of the past. Mathematically, it can be presented as:

$$P(X_t = j | X_0 = i_0, X_1 = i_1, \dots, X_{t-1} = i_{t-1}) = P(X_t = j | X_{t-1} = i_{t-1}). \tag{4}$$

Definition 6 (Ching & Ng, 2006): State i is said to be reachable from state j if $P_{ij}^{(n)} > 0$ and for some $n > 0$. This means that starting from state j , it is possible to enter state i in finite number of transitions.

Definition 7 (Ching & Ng, 2006): State i and state j are said to communicate if state i and state j are reachable from each other.

Methodology

Fuzzy Time Series-Markov Chain

Tsaur (2012) first developed this method by using the Song and Chissom (1993; 1994) and Chen (1996) concepts. The flowchart of fuzzy time series-Markov chain is presented in Figure 1. Then, the algorithm of fuzzy time series-Markov chain is shown as the following:

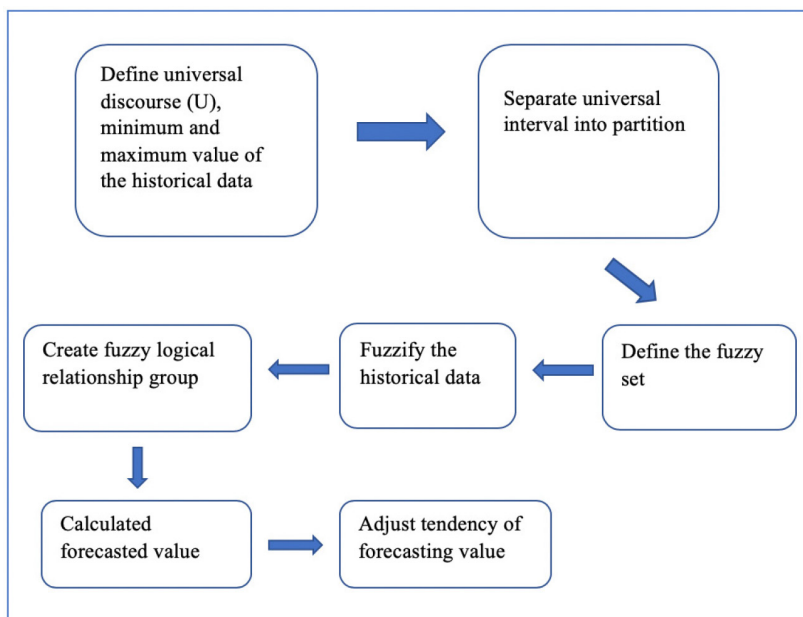


Figure 1: Flowchart of fuzzy time series-Markov chain. NB: Calculate forecasted value

Step 1: Define the universal discourse U , minimum data (D_{\min}) and maximum data (D_{\max}) for the historical data. The universal discourse is defined as:

$$U = [D_{\min} - D_1, D_{\max} + D_2]. \tag{5}$$

Step 2: Separate U into several equal intervals. The universal of discourse is then divided into 7 equal-length intervals with $l = 0.1003$.

Step 3: Define the fuzzy set using Eq. (6):

$$\begin{aligned} A_1 &= \frac{a_{11}}{u_1} + \frac{a_{12}}{u_2} + \dots + \frac{a_{1n}}{u_n}, \\ A_2 &= \frac{a_{21}}{u_1} + \frac{a_{22}}{u_2} + \dots + \frac{a_{2n}}{u_n}, \\ &\vdots \\ A_k &= \frac{a_{k1}}{u_1} + \frac{a_{k2}}{u_2} \dots + \frac{a_{kn}}{u_n}, \end{aligned} \tag{6}$$

where $A_i, i = 1, \dots, k$ are the fuzzy sets which are represented as linguistic values of the linguistic variables.

Step 4: Fuzzify the historical data to fuzzy set A_i .

Step 5: Create fuzzy logical relationship group.

Step 6: Calculate forecasting value, $f(t)$ according to the rules.

Rule 1: If $Y(t-1) = A_j$ and it is one-to-one fuzzy logical relationship, $A_j \rightarrow A_k$, the forecasted future value, $f(t)$ is calculated by using Eq. (7).

$$f(t) = m_k \tag{7}$$

Rule 2: If $Y(t-1) = A_j$ and it is one-to-many fuzzy logical relationship, for example $A_j \rightarrow A_1, A_2, \dots, A_n, j = 1, 2, \dots, n$, the forecasted future value, $f(t)$ is:

$$\begin{aligned} F(t) &= m_1 P_{j1} + m_2 P_{j2} + \dots + m_{j-1} P_{j(j-1)} + Y(t-1) \\ &P_{jj} + m_{j+1} P_{j(j+1)} + \dots + m_n P_{jn}, \end{aligned} \tag{8}$$

where,

- m_i : midpoint of interval u_i
- P_{jn} : probability from state j to state n

Step 7: Adjust the tendency of the forecasting value according to the rules.

Rule 1: If A_i communicates with A_j and makes an increasing transition, where $Y(t-1) = A_i, Y(t) = A_j, (A_i \rightarrow A_j, i > j)$, then the adjusted trend value will be:

$$D_{t1} = \frac{l}{2}. \tag{9}$$

Rule 2: If A_i communicates with A_j and makes a decreasing transition, where $Y(t-1) = A_i, Y(t) = A_j, (A_i \rightarrow A_j, i > j)$, then the adjusted trend value will be:

$$D_{t1} = -\frac{l}{2}. \tag{10}$$

Rule 3: If $Y(t-1) = A_i$ and makes a jump-forward transition at time t where $Y(t) = A_{i+s}, (1 \leq s \leq n - i)$, then the adjusted trend value will be:

$$D_{t2} = \frac{l}{2}(s) \tag{11}$$

Rule 4: If $Y(t-1) = A_i$ and makes a jump-backward transition at time t where $Y(t) = A_{i-v}, (1 \leq v \leq i)$ then the adjusted trend value will be:

$$D_{t2} = -\frac{l}{2}(v). \tag{12}$$

Step 8: Calculate the adjusted forecasting values.

$$F'(t) = F(t) \pm D_{t1} \pm D_{t2}. \tag{13}$$

Performance Evaluation of Model

The performance of the proposed model is evaluated by using the following Mean Absolute Percentage Error (MAPE) formula:

$$MAPE = \frac{1}{n} \sum_{t=1}^n \frac{|Y(t) - F'(t)|}{Y(t)} \times 100\%. \tag{14}$$

Analysis on Case Study

In this study, the historical data of exchange rate between MYR and SGD from year 2015 to year 2019 were collected from official website of Bank Simpanan Nasional (www.bnm.gov.my). The proposed model fuzzy time series-Markov chain is used to forecast the exchange rate by using Microsoft Excel.

Step 1: The universal discourse, U in this study is defined as $U = [2.4570, 3.1590]$.

Step 2: The universal of discourse is then divided into 7 equal-length intervals with $l = 0.1003$. Then, the interval is defined as below:

- $u_1 = [2.4570, 2.5572]$,
- $u_2 = [2.5573, 2.6575]$,
- $u_3 = [2.6576, 2.7578]$,
- $u_4 = [2.7579, 2.8581]$,
- $u_5 = [2.8582, 2.9583]$,
- $u_6 = [2.9584, 3.0586]$,
- $u_7 = [3.0587, 3.1589]$.

Step 3: Consider the forecasting of interval in triangular membership function, the fuzzy sets are defined as:

$$A_1 = \frac{1}{u_1} + \frac{0.5}{u_2} + \frac{0}{u_3} + \frac{0}{u_4} + \frac{0}{u_5} + \frac{0}{u_6} + \frac{0}{u_7},$$

$$A_2 = \frac{0.5}{u_1} + \frac{1}{u_2} + \frac{0.5}{u_3} + \frac{0}{u_4} + \frac{0}{u_5} + \frac{0}{u_6} + \frac{0}{u_7},$$

$$A_3 = \frac{0}{u_1} + \frac{0.5}{u_2} + \frac{1}{u_3} + \frac{0.5}{u_4} + \frac{0}{u_5} + \frac{0}{u_6} + \frac{0}{u_7},$$

$$A_4 = \frac{0}{u_1} + \frac{0}{u_2} + \frac{0.5}{u_3} + \frac{1}{u_4} + \frac{0.5}{u_5} + \frac{0}{u_6} + \frac{0}{u_7},$$

$$A_5 = \frac{0}{u_1} + \frac{0}{u_2} + \frac{0}{u_3} + \frac{0.5}{u_4} + \frac{1}{u_5} + \frac{0.5}{u_6} + \frac{0}{u_7},$$

$$A_6 = \frac{0}{u_1} + \frac{0}{u_2} + \frac{0}{u_3} + \frac{0}{u_4} + \frac{0.5}{u_5} + \frac{1}{u_6} + \frac{0.5}{u_7},$$

$$A_7 = \frac{0}{u_1} + \frac{0}{u_2} + \frac{0}{u_3} + \frac{0}{u_4} + \frac{0}{u_5} + \frac{0.5}{u_6} + \frac{1}{u_7}.$$

Step 4: The fuzzified exchange rate is matched with its equivalent fuzzy sets. The result is shown in Table 1.

Table 1: The Fuzzification of Data

Month	Exchange rate	Fuzzy data	Month	Exchange rate	Fuzzy data
Jan-15	2.6792	A_3	Jul-17	3.1282	A_7
Feb-15	2.6524	A_2	Aug-17	3.1482	A_7
Mar-15	2.6726	A_3	Sep-17	3.1193	A_7
Apr-15	2.6923	A_3	Oct-17	3.1085	A_7
May-15	2.6999	A_3	Nov-17	3.0769	A_7
Jun-15	2.7781	A_4	Dec-17	3.0284	A_6
Jul-15	2.7934	A_4	Jan-18	2.9925	A_6
Aug-15	2.9042	A_5	Feb-18	2.9635	A_6
Sep-15	3.0458	A_6	Mar-18	2.9690	A_6
Oct-15	3.0432	A_6	Apr-18	2.9555	A_5
Nov-15	3.0506	A_6	May-18	2.9622	A_6
Dec-15	3.0393	A_6	Jun-18	2.9687	A_6
Jan-16	3.0323	A_6	Jul-18	2.9704	A_6
Feb-16	2.9792	A_6	Aug-18	2.9896	A_6

Mar-16	2.9655	A_6	Sep-18	3.0209	A_6
Apr-16	2.8904	A_5	Oct-18	3.0154	A_6
May-16	2.9535	A_5	Nov-18	3.0433	A_6
Jun-16	3.0140	A_6	Dec-18	3.0453	A_6
Jul-16	2.9755	A_6	Jan-19	3.0350	A_6
Aug-16	2.9901	A_6	Feb-19	3.0092	A_6
Sep-16	3.0233	A_6	Mar-19	3.0125	A_6
Oct-16	3.0177	A_6	Apr-19	3.0340	A_6
Nov-16	3.0737	A_7	May-19	3.0438	A_6
Dec-16	3.1070	A_7	Jun-19	3.0540	A_6
Jan-17	3.1214	A_7	Jul-19	3.0322	A_6
Feb-17	3.1422	A_7	Aug-19	3.0242	A_6
Mar-17	3.1583	A_7	Sep-19	3.0329	A_6
Apr-17	3.1521	A_7	Oct-19	3.0542	A_6
May-17	3.0933	A_7	Nov-19	3.0545	A_6
Jun-17	3.0909	A_7	Dec-19	2.4573	A_1

Step 5: The fuzzy logical relationship in this study is determined and shown in Figure 2

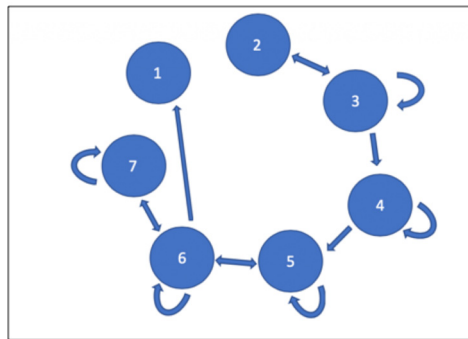


Figure 2: Transition process of the exchange rate

Step 6: The forecasting value, $F(t)$ is calculated according to the rules. The forecasting value for Feb-15 in this study is shown as below.

$$\begin{aligned}
 F(\text{Feb} - 15) &= \frac{1}{4}m_2 + \frac{1}{2}(F(t - 1)) + \frac{1}{4}(m_4) \\
 &= \frac{1}{4}(2.6074) + \frac{1}{2}(F(t - 1)) + \frac{1}{4}(2.8080) \\
 &= \frac{1}{4}(2.6074) + \frac{1}{2}(2.6792) + \frac{1}{4}(2.8080) \\
 &= 2.6934.
 \end{aligned}$$

Step 7: The tendency of the forecasting value is adjusted according to the rules. For example, the adjustment for Feb-15 is obtained as follows:

$$\begin{aligned}
 \text{Adjustment for Feb} - 15 &= -D_{t1} \\
 &= -\frac{l}{2} \\
 &= -\frac{0.1003}{2} \\
 &= -0.0501.
 \end{aligned}$$

Step 8: The adjusted forecasting values, $F'(t)$ are obtained and presented in Table 2.

Results and Discussion

The result of the forecasting value using fuzzy time series-Markov chain is shown in Table 2.

Table 2: Forecasting value of the exchange rate between MYR and SGD

Month	Actual Exchange rate	$F'(t)$	Month	Actual Exchange rate	$F'(t)$
Jan-15	2.6792		Jul-17	3.1282	3.0845
Feb-15	2.6524	2.6433	Aug-17	3.1482	3.1190
Mar-15	2.6726	2.7077	Sep-17	3.1193	3.1375
Apr-15	2.6923	2.6901	Oct-17	3.1085	3.1108
May-15	2.6999	2.7000	Nov-17	3.0769	3.1008
Jun-15	2.7781	2.7539	Dec-17	3.0284	3.0215
Jul-15	2.7934	2.8432	Jan-18	2.9925	3.0089
Aug-15	2.9042	2.9010	Feb-18	2.9635	2.9771
Sep-15	3.0458	3.0326	Mar-18	2.9690	2.9515
Oct-15	3.0432	3.0244	Apr-18	2.9555	2.9062
Nov-15	3.0506	3.0220	May-18	2.9622	3.0454
Dec-15	3.0393	3.0286	Jun-18	2.9687	2.9503
Jan-16	3.0323	3.0186	Jul-18	2.9704	2.9561
Feb-16	2.9792	3.0124	Aug-18	2.9896	2.9575
Mar-16	2.9655	2.9653	Sep-18	3.0209	2.9746
Apr-16	2.8904	2.9031	Oct-18	3.0154	3.0023
May-16	2.9535	2.9790	Nov-18	3.0433	2.9974
Jun-16	3.0140	3.0449	Dec-18	3.0453	3.0222
Jul-16	2.9755	2.9962	Jan-19	3.0350	3.0239
Aug-16	2.9901	2.9620	Feb-19	3.0092	3.0148
Sep-16	3.0233	2.9750	Mar-19	3.0125	2.9919
Oct-16	3.0177	3.0044	Apr-19	3.0340	2.9948
Nov-16	3.0737	3.0496	May-19	3.0438	3.0139
Dec-16	3.1070	3.0687	Jun-19	3.0540	3.0225
Jan-17	3.1214	3.0994	Jul-19	3.0322	3.0316
Feb-17	3.1422	3.1127	Aug-19	3.0242	3.0123
Mar-17	3.1583	3.1319	Sep-19	3.0329	3.0052
Apr-17	3.1521	3.1468	Oct-19	3.0542	3.0129
May-17	3.0933	3.1411	Nov-19	3.0545	3.0318
Jun-17	3.0909	3.0868	Dec-19	2.4573	2.7814

Performance Evaluation of Model

The performance of the proposed model is evaluated by using the Mean Absolute Percentage Error (MAPE) in Eq. (14).

$$\begin{aligned}
 MAPE &= \frac{1}{n} \sum_{t=1}^n \frac{|Y(t) - F'(t)|}{Y(t)} \times 100\% \\
 &= \frac{1}{59} \sum_{t=1}^{59} \frac{|Y(t) - F'(t)|}{Y(t)} \times 100\% \\
 &= \frac{1}{59} \times 0.5838 \times 100\% \\
 &= 0.9895\% .
 \end{aligned}$$

According to Lewis (1982), the MAPE value below 10% is considered as highly accurate. The fuzzy time series-Markov chain in exchange rate forecasting obtained a value of 0.9895%. Therefore, the proposed method is highly accurate for exchange rate forecasting in this study.

Comparison of Performance Evaluation

Exchange rate forecasting between MYR and SGD is performed by using fuzzy time series to provide a comparison model. The actual value and the forecasting value of the exchange rate using fuzzy time series and fuzzy time series-Markov chain are depicted in Figure 3.

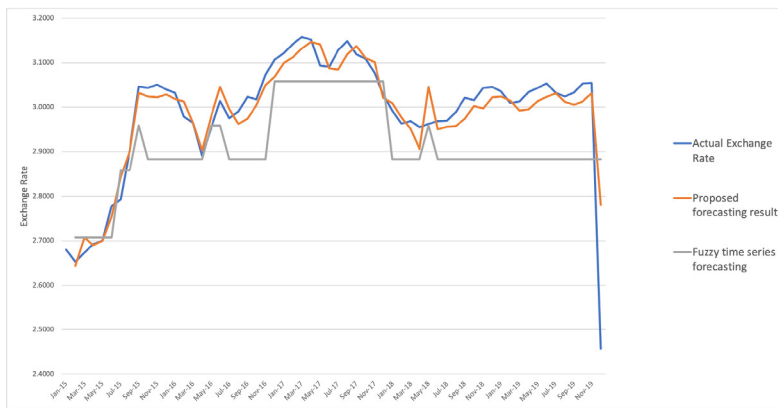


Figure 3: Comparisons in Exchange Rate Forecasting

From Figure 3, the comparison graph shows that the forecasting result of the proposed model is closer to the actual exchange rate.

The MAPE value for fuzzy time series forecasting is also calculated. The result is shown in Table 3.

Table 3: Comparison of MAPE Values

Forecasting methods	MAPE value
Fuzzy Time Series	3.4306 %
Fuzzy Time Series Markov Chain	0.9895%

Conclusion

This study explores the benefit of fuzzy time series-Markov chain in the forecasting of exchange rate between Malaysian Ringgit (MYR) and Singapore Dollar (SGD). The results indicated that the integrated model has a better

forecasting accuracy compared to the classical models such as fuzzy time series and Markov chain. Both fuzzy time series model and fuzzy time series-Markov chain model give a satisfied performance in the forecasting of exchange rate. However, the fuzzy time series-Markov chain

has a smaller MAPE value which means higher accuracy in exchange rate forecasting.

For a further study, the advanced software packages such as MATLAB, SAS, and MINITAB can be applied to the proposed model to simulate the results conveniently and effectively. Besides, further exploration such as the use of higher-order fuzzy time series model or higher-order Markov chain model is needed to aim for a better performance in exchange rate forecasting.

Acknowledgements

This work is a part of the final year project under the BSc of Financial Mathematics program, FTKKI, UMT. The authors are grateful to anonymous referees for comments and suggestions that helped improve the paper.

References

- Adesokan, I., Ngare, P., & Kilishi, A. (2017). Analyzing expected returns of a stock using the Markov chain model and the capital asset pricing model. *Applied Mathematical Sciences*, 11(56), 2777-2788.
- Adhikari, R., & Agrawal, R. K. (2013). *An introductory study on time series modeling and forecasting*. LAP Lambert Academic Publishing, Germany.
- Ali, O. A., Ali, A. Y., & Sumait, B. S. (2015). Comparison between the effects of different types of membership functions on fuzzy logic controller performance. *International Journal of Emerging Engineering Research and Technology*, 3(3), 76-83.
- Avrachenkov, K. E., & Sanchez, E. (2002). Fuzzy Markov Chains and Decision-Making. *Fuzzy Optimization and Decision Making*, 1, 143-159.
- Boiroju, N. K., Rao, M. V., & Reddy, M. K. (2011). Forecasting Foreign Exchange Rates Using Fuzzy Time Series. *International Journal of Statistics and Systems*, 6(1), 153-161.
- Chang, S. S. L., & Zadeh, L. A. (1972). On fuzzy mapping and control. *IEEE Transactions on Systems, Man, and Cybernetics*, 2, 30-34.
- Chatfield, C. (2000). *Time series forecasting*. Chapman and Hall, London.
- Chen, S. M. (1996). Forecasting enrollments based on fuzzy time series. *Fuzzy Sets and Systems*, 81, 311-319.
- Ching, W. K., & Ng, M. (2006). *Markov chains: Models, algorithms and applications*. Springer, New York.
- Erdogan, O., & Goksu, A. (2014). Forecasting Euro and Turkish Lira exchange rates with Artificial Neural Networks (ANN). *International Journal of Academic Research in Accounting, Finance and Management Sciences*, 4(4), 307-316.
- He, Z. C., & Jiang, W. (2018). A new belief Markov chain model and its application in inventory prediction. *International Journal of Production Research*, 56(8), 2800-2817.
- Islam, M. D., & Hossain, E. (2020). Foreign Exchange Currency Rate Prediction using a GRU-LSTM Hybrid Network. *Soft Computing Letters*, 1-20.
- Lewis, C. D. (1982). *Industrial and business forecasting methods: A practical guide to exponential smoothing and curvefitting*. Butterworth Scientific, London.
- Li, G., & Xiu, B. (2014). Fuzzy Markov chains based on the fuzzy transition probability. *The 26th Chinese Control and Decision Conference*, 4351-4356.
- Madi, E., Naim, S., Yaafar, A., Yaakob, A. M., & Yusoff, B. (2020). Agreement Matrix Based on Fuzzy Decision Making to Rank Ship Berthing Criteria. *International Journal of Engineering Trends and Technology*, 68(12), 31-36.
- Meyer, A., & Zimmermann, H. -J. (2011). Applications of Fuzzy Technology in Business Intelligence. *International Journal of Computers, Communications & Control*, 4(3), 428-441.

- Reid, R. D., & Sanders, N. R. (2010). *Operations management an integrated approach*, 4th edition. Wiley, Hoboken, New Jersey.
- Sahin, A., Kumbasar, T., Yesil, E., Dodurka, M. F., & Karasakal, O. (2014). An approach to represent time series forecasting via fuzzy numbers. *2nd International Conference on Artificial Intelligence, Modelling and Simulation*, 51-56.
- Seneta, E. (2006). Markov and the the creation of Markov chains. *MAM 2006: Markov Anniversary Meeting*. Bosen Books, Raleigh, North Carolina, 1-20.
- Song, Q., & Chissom, B. S. (1993). Forecasting enrolments with fuzzy time series-Part I. *Fuzzy Sets and Systems*, 54(1), 1-9.
- Song, Q., & Chissom, B. S. (1994). Forecasting enrollments with fuzzy time series-Part II. *Fuzzy Sets and Systems*, 62(1), 1-8.
- Tayal, D., Sonawani, S., Ansari, G., & Gupta, C. (2011). Fuzzy time series forecasting of low dimensional numerical data. *International Journal of Engineering Research and Applications*, 2(1), 132-135.
- Tsaur, R. C. (2012). A fuzzy time series-Markov chain model with an application to forecast the exchange rate between the Taiwan And US Dollar. *International Journal of Innovative Computing, Information and Control*, 8, 4931-4942.
- Uzun, B., & Kiral, E. (2017). Application of Markov chains-fuzzy states to gold price. *Procedia Computer Science*, 120, 365-371 .
- Zadeh, L. A. (1965). Fuzzy sets. *Information and Control*, 8, 338-353.
- Zadeh, L. A. (1973). Outline of a new approach to the analysis of complex systems and decision processes. *IEEE Transactions On Systems, Man, and Cybernetics*, Smc-3(1), 28-44.
- Zhang, D., & Zhang X. M. (2009). Study on forecasting the stock market trend based on stochastic analysis method. *International Journal of Business and Management*, 4(6).

