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AN APPLICATION OF BOUNDED AUTOCATALYTIC SET WITH SHANNON ENTROPY FOR DESKTOP COMPUTER SELECTION

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ABSTRACT

This article proposes a Multi-criteria Decision-making (MCDM) approach by integrating the Bounded Autocatalytic Set (BACS) method with Shannon entropy for desktop computer selection. Existing Graph Theory and Matrix Approach (GTMA)-based decision models often neglect the weighting of criteria, which may lead to less reliable ranking outcomes. To address this limitation, the proposed method incorporates Shannon entropy to objectively determine attribute weights before applying the BACS framework for ranking alternatives. The integration provides methodological advantages by combining a graphical representation of attribute interrelationships with entropy-based weighting, resulting in a more systematic and informative decision-making process. A numerical example involving five desktop computer models (M1–M5), evaluated across five attributes, is presented to demonstrate the applicability of the method. The results show that Model M3 achieves the highest permanent index value of 8.0396, indicating that it is the most preferred of the models being evaluated. The ranking results are further validated through comparison with the Graph Theory and Matrix Approach (GTMA) and a hybrid Analytic Hierarchy Process (AHP) – Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) method. Spearman correlation coefficients of 0.9 indicate a strong agreement between the proposed method and existing approaches, thereby confirming the reliability of the ranking outcomes. The findings demonstrate that the integration of BACS and Shannon entropy provides an effective and robust framework for solving MCDM problems.

2020 Mathematics Subject Classification:

Introduction

Bounded Autocatalytic Set (BACS) is a directed graph in which there is a directed edge connecting a pair of vertices in both directions [1]. Generally, BACS is a non-loop subgraph G (V, E) in which each vertex $v \in V$ has exactly $(n - 1)$ incoming links and $(n - 1)$ outgoing links for $|V| = n$. Several analyses of the basic properties of the BACS graph related to paths and cycles are investigated [1].

The BACS graph is inspired by the digraph representation used to visualise the interrelation among attributes in Graph Theory and Matrix Approach (GTMA) [2], which is widely applied to model the interrelationship among attributes in complex systems. Due to its graphical structure, BACS has the capability to represent interactions between attributes and to support analytical decision-making processes. Examples of BACS graph with two vertices, three vertices, and four vertices are shown in Figure 1.

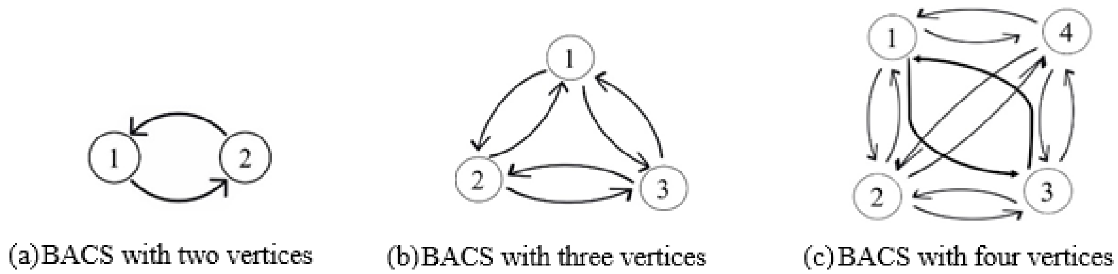

Figure 1: Some examples of BACS graph

Multi-criteria Decision-making (MCDM) problems frequently involve multiple conflicting attributes that must be evaluated simultaneously. Several decision-making approaches such as Graph Theory and Matrix Approach (GTMA), Analytic Hierarchy Process (AHP), and Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) have been widely applied in solving such problems. However, studies show that the conventional GTMA method is often implemented without incorporating an objective weighting mechanism for the criteria [12–22]. The absence of appropriate weighting may reduce the reliability and accuracy of the final ranking results because different attributes may contribute unequally to the decision-making process. Therefore, incorporating an effective weighting technique is essential to improve the robustness of the decision model [23–24].

To address the limitations of existing approaches, this study integrates the Bounded Autocatalytic Set (BACS) method with Shannon entropy to develop a more robust MCDM framework. Shannon entropy, also known as informational entropy [3], was originally used to explain irreversible motion in thermodynamics, However, it was later found to be effective in addressing decision-making problems [4]. In this study, the Shannon entropy formula is extended into the entropy weight method for calculating the weights of the criteria based on the degree of information contained in the data, while BACS is used to represent the relationships between attributes and to evaluate the alternatives through the permanent function of the matrix.

One of the typical MCDM problems involving multiple attributes that need to be evaluated simultaneously is desktop computer selection. Multiple attributes such as processor performance, brand reliability, screen size, storage capacity, and RAM must be evaluated simultaneously. Thus, selecting the most suitable desktop computer requires a systematic evaluation framework that can consider the relationships between attributes and their relative importance. Hence, the desktop computer selection problems provide a suitable context to demonstrate the applicability of the proposed decision-making approach.

The main contributions of this study are as follows:

i. To propose an integration of BACS with Shannon entropy for solving MCDM problems;
ii. To incorporate objective criteria weighting into the BACS framework; and,
iii. To demonstrate the applicability and reliability of the proposed method through a desktop computer selection case study with validation using Spearman correlation analysis. The methodology of the proposed approach is presented in the following section.

Proposed Method

The procedures of BACS with Shannon entropy method are shown in Figure 2.

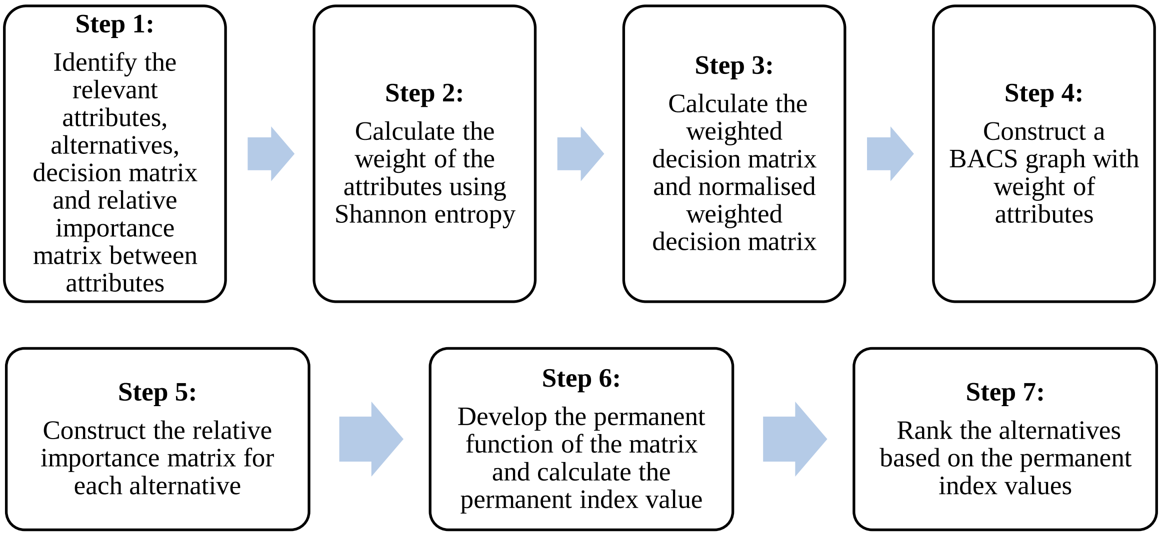

Figure 2: The procedure of BACS and Shannon entropy

The framework consists of seven main steps starting from the identification of alternatives and attributes, followed by the calculation of attribute weights using Shannon entropy. The weighted decision matrix is then constructed and normalised to eliminate scale differences between attributes. Subsequently, a BACS graph is developed to represent the relationships among attributes. The relative importance matrix for each alternative is then constructed based on the normalised weighted values. Finally, the permanent function of the matrix is evaluated to obtain the permanent index value, which is used to rank the alternatives. This structured procedure ensures that both attribute weighting and attribute interrelationships are incorporated in the decision-making process. The detailed procedures are given as follows.

Step 1: Identify the Relevant Attributes, Alternatives, Decision Matrix, and Relative Importance Matrix Between Attributes

In Step 1, the relevant attributes, alternatives, decision matrix, and relative importance matrix are identified from Mitra and Goswami [25]. Let A = {A1, A2,..., Am} denote the set of alternatives and C = {C1, C2,..., Cn} denote the set of evaluation attributes. The decision matrix, denoted by D = [xij], represents the performance value of alternative Ai with respect to attribute Cj, where i = 1, 2,..., m and j = 1, 2,..., n. Thus, each element xij indicates the measured performance score of alternative Ai under attribute Cj. The decision matrix, D is represented in Equation (1).

$$D = \begin{bmatrix} x_{11} & \cdots & x_{1j} & \cdots & x_{1n} \\ x_{21} & \cdots & x_{2j} & \cdots & x_{2n} \\ x_{31} & \cdots & x_{3j} & \cdots & x_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x_{m1} & \cdots & x_{mj} & \cdots & y_{mn} \end{bmatrix}. \tag{1}$$

In addition, the relative importance matrix, denoted by $B = [b_{ij}]$, represents the pairwise importance relationships between attributes. The diagonal elements $b_{ii} = y_i$ represent the attribute values derived from the normalised weighted decision matrix, while the off-diagonal elements b_{ij}; $i \neq j$ represent the relative importance of attribute C_i compared with attribute C_j as determined by the decision maker. The matrix B is a square matrix of order n. The relative importance matrix of attribute B is denoted in Equation (2).

$$B = \begin{bmatrix} y_1 & b_{12} & b_{13} & \cdots & b_{1n} \\ b_{21} & y_2 & b_{23} & \cdots & b_{2n} \\ b_{31} & b_{32} & y_3 & \cdots & b_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & b_{n3} & \cdots & y_n \end{bmatrix}. \tag{2}$$

Step 2: Calculate the Weight of the Attributes using Shannon Entropy

In this step, the data obtained in Step 1 is used to calculate the weight of the attributes. First, the normalised decision matrix, $D^* = (p_{ij} : i, j = 1, 2, ..., n)$ is calculated to eradicate the differences between units and scales using Equation (3) where:

$$p_{ij} = \frac{x_{ij}}{\sum_{i=1}^{n} x_{ij}} \tag{3}$$

Then, the value of entropy, h_i is calculated using Equation (4) where h_0 represents the entropy constant and is equal to $(Ln\,(m))^{-1}$ where m represents the number of alternatives and $(p_{ij})\square Ln\,(p_{ij})$ is described as 0 if $p_{ij} = 0$.

$$h_P = -h_0 \left(\sum_{i=1}^{5} p_{ij} \times \ln(p_{ij}) \right) \tag{4}$$

Subsequently, the degree of the diversification, d_i is computed using Equation (5).

$$d_i = 1 - h_i, i = 1, 2, ..., n \tag{5}$$

The degree of importance of attribute i, w_i is calculated using the Equation (6).

$$w_i = \frac{d_i}{\sum_{s=1}^{n} d_s}, i = 1, 2, ..., n \tag{6}$$

Step 3: Calculate the Weighted Decision Matrix and Normalised Weighted Decision Matrix

In this step, the weighted decision matrix, v_{ij} is calculated using Equation (7).

$$v_{ij} = w_i . p_{ij}, i = 1, 2, 3, ..., n; j = 1, 2, 3, ..., m. \tag{7}$$

Moreover, the normalised weighted decision matrix, $v_{ij}*$ are determined using Equation (8).

$$v_{ij}* = \frac{v_{ij}}{\max_{j=1} x_{ij}}, i = 1, 2, 3, ..., n. \tag{8}$$

Step 4: Construct a BACS Graph with Weight of Attributes

In this step, the BACS graph for a given selection problem is constructed where each of the attributes has their respective weights as obtained in Step 2. Besides, the BACS graph consists of a group of vertices and a group of directed edges. The vertices denote the i^{th} selection attributes and the directed edges denote the relative importance between the attributes. For instance, if vertex i has relative importance over vertex j, then a directed edge is drawn from vertex i to vertex j which is represented as e_{ij}. Similarly, if vertex j has relative importance over vertex i, then a directed edge is drawn from vertex j to vertex i which is represented as e_{ji}. Example of BACS graph is illustrated in Figure 3.

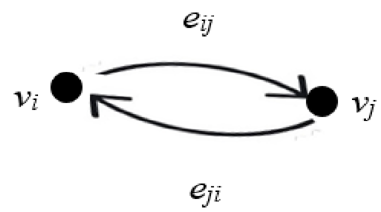

Figure 3: A BACS graph

Step 5: Construct the Relative Importance Matrix for Each Alternative

Subsequently, in this step, the relative importance matrix, B for each alternative is constructed. The matrix B is a square matrix that consists of diagonal elements, y_i and off-diagonal elements, b_{ij}. The relative importance matrix for each attribute is formed using the relative importance matrix obtained in Step 1 and the values of the normalised weighted decision matrix, $v_{ij}*$ is used as the diagonal elements.

Step 6: Develop the Permanent Function of the Matrix and Calculate the Permanent Index Value

Here, in this step, the permanent function of the matrix for each alternative is developed. The permanent function as mentioned in Rao [11] and Agrawal *et al.* [26] has $(M+1)$ group for the matrix of $M \times M$ which signifies the existence of attributes and the relative importance loops. The permanent function is denoted as Per(A) and known as index score for the alternative [26] as shown below.

$$
\begin{aligned}
&\mathrm{Per(A)} \\
&= \prod_{i=1}^{M} Si + \sum_i \sum_j \sum_k \ldots \sum_m (a_{ij}a_{ji}) R_k R_l \ldots R_m \\
&\quad + \sum_i \sum_j \sum_k \ldots \sum_m (a_{ij}a_{ji}a_{ki} + a_{ik}a_{kj}a_{ji}) R_l R_m \ldots R_m \\
&\quad + \left(\sum \sum \sum \ldots \sum (a_{ij}a_{ji})(a_{kl}a_{lk}) R_m R_n \ldots R_m + \sum_i \sum_j \sum_k \ldots \sum_m (a_{ij}a_{jk}a_{kl}a_{li} + a_{il}a_{lk}a_{kj}a_{ji}) R_m R_n \ldots R_m \right) \\
&\quad + \left(\sum_i \sum_j \sum_k \ldots \sum_m (a_{ij}a_{ji})(a_{kl}a_{lm}a_{mk} + a_{km}a_{ml}a_{lk}) R_n R_o \ldots R_m + \sum_i \sum_j \sum_k \ldots \sum_m (a_{ij}a_{jk}a_{kl}a_{lm}a_{mi} + a_{im}a_{ml}a_{lk}a_{kj}a_{ji}) R_n R_o \ldots R_m + \ldots \right)
\end{aligned}
$$

The permanent function of the matrix involves a large number of multiplicative combinations of matrix elements, particularly when the number of attributes increases. Manual computation of the permanent function becomes computationally intensive and prone to human error. Therefore, Matrix Laboratory (MATLAB) software is employed to calculate the permanent index values efficiently. MATLAB provides a reliable numerical computing environment with high computational accuracy and built-in matrix manipulation capabilities, making it suitable for handling complex matrix operations required in evaluating the permanent function. The use of MATLAB also improves computational efficiency and reproducibility of the results.

Step 7: Rank the Alternatives Based on the Permanent Index Values

In this last step, the alternatives are ranked based on the permanent index value where the higher the permanent index value, the higher the rank of the alternative. In other words, the alternative with the highest permanent index value is recognised as the best choice.

Numerical Example

To demonstrate the applicability of the proposed method, the dataset from Mitra and Goswami [25] is adopted. However, several modifications were performed to ensure compatibility with the proposed framework. Specifically, the original AHP based evaluation scores were transformed into a normalised BACS scale within the interval [0,1] and the decision matrix was restructured accordingly. These modifications are necessary to enable integration with the BACS representation and entropy-based weighting procedure.

Step 1

In this step, the relevant attributes, alternatives, decision matrix, and relative importance matrix between attributes are identified. In the desktop computer selection, there are five alternatives considered for the evaluation such as Model-1 computer (M1), Model-2 computer (M2), Model-3 computer (M3), Model-4 computer (M4), and Model-5 computer (M5). Moreover, all five alternatives are evaluated based on five attributes namely processor (P), brand (B), screen size (SS), hard disk capacity (HDC), and RAM (R). The decision matrix for the desktop computer selection problem is presented in Table 1.

Table 1: Decision matrix for desktop computer selection problem

Alternative	**Attributes**				
	Processor (P)	**Brand (B)**	**Screen Size (SS)**	**Hard Disk Capacity (HDC)**	**RAM (R)**
M1	4	7	2	2	3
M2	4	5	3	5	7
M3	7	4	9	9	7
M4	3	9	9	5	2
M5	7	1	5	5	7

To convert the AHP scale (1-9) into the BACS scale (0-1), a rational linear transformation is applied. The transformation is defined as:

$$x' = \frac{x-1}{9-1} = \frac{x-1}{8}$$

where $x \in [1,9]$ represents the original AHP scale and $x' \in [0,1]$ denotes the corresponding BACS scale. This transformation preserves the relative ordering of preferences while standardising the scale for subsequent analysis. The linear transformation of (1-9) scale to (0-1) scale is depicted in Table 2.

Table 2: Linear transformation of 1 to 9 scale

Preference Scale	**AHP**	**BACS**	
		a_{ij}	**1 -** a_{ij}
Equal importance	1	0.000	1.000
Weak	2	0.125	0.875
Moderate importance	3	0.250	0.750
Moderate plus	4	0.375	0.625
Strong importance	5	0.500	0.500
Strong plus	6	0.625	0.375
Very strong importance	7	0.750	0.250
Very very strong	8	0.875	0.125
Extreme importance	9	1.000	0.000

Furthermore, the decision matrix in Table 3 is formed after a linear transformation is carried out.

Table 3: Decision matrix after linear transformation

Alternative	**Attributes**				
	Processor (P)	**Brand (B)**	**Screen Size (SS)**	**Hard Disk Capacity (HDC)**	**RAM (R)**
M1	0.375	0.750	0.125	0.125	0.250
M2	0.375	0.500	0.250	0.500	0.750
M3	0.750	0.375	1.000	1.000	0.750
M4	0.250	1.000	1.000	0.500	0.125
M5	0.750	0.125	0.500	0.500	0.750

Subsequently, the normalisation of the attribute data given in decision matrix in Table 3 is performed using Equation (3). The normalised decision matrix is presented in Table 4.

Table 4: Normalised decision matrix

Alternative	**Processor (P)**	**Brand (B)**	**Screen Size (SS)**	**Hard Disk Capacity (HDC)**	**RAM (R)**
M1	0.1500	0.2727	0.0435	0.0476	0.0952
M2	0.1500	0.1818	0.0870	0.1905	0.2857
M3	0.3000	0.1364	0.3478	0.3810	0.2857
M4	0.1000	0.3636	0.3478	0.1905	0.0476
M5	0.3000	0.0455	0.1739	0.1905	0.2857

Next, consider the relative importance matrix between attributes as given in Table 5.

Table 5: Relative importance matrix between attributes

	P	**B**	**SS**	**HDC**	**R**
P	X_1	0.750	0.500	0.250	0.500
B	0.250	X_2	0.375	0.750	0.875
SS	0.500	0.625	X_3	0.250	0.250
HDC	0.750	0.250	0.750	X_4	0.250
R	0.500	0.125	0.750	0.750	X_5

Step 2

In this step, the weights of the attributes are calculated using Shannon entropy-based on Equations 4, 5, and 6. For example, from Table 4, the normalised values for attribute P are: (0.1500,0.1500,0.3000,0.1000,0.3000). The entropy value for attribute P is computed using Equation (4):

$$\begin{aligned} h_P &= -h_0 \sum_{i=1}^{5} p_{ij} \times \ln(p_{ij}) \\ &= -\frac{1}{\ln(5)}\left[0.15\ln(0.15) + 0.15\ln(0.15) + 0.3\ln(0.3) + 0.1\ln(0.1) + 0.3\ln(0.3)\right] \\ &= 0.9455 \end{aligned}$$

Subsequently, the degree of diversification is:

$$d_P = 1 - 0.9455 = 0.0545$$

Finally, the normalised weight of attribute P is:

$$w_P = \frac{d_p}{\sum d_P + d_B + d_{SS} + d_{HDC} + d_R} = \frac{0.0545}{0.0545 + 0.1026 + 0.1379 + 0.0927 + 0.1036} = 0.1109$$

The same procedure is applied to all other attributes, and the results are summarised in Table 6.

Table 6: The values of h_i, d_i, and w_i

Alternative\ $p_{ij} \times \ln(p_{ij})$	**P**	**B**	**SS**	**HDC**	**R**
M1	-0.2846	-0.3543	-0.1363	-0.1450	-0.2239
M2	-0.2846	-0.3100	-0.2124	-0.3159	-0.3579
M3	-0.3612	-0.2717	-0.3673	-0.3676	-0.3579
M4	-0.2303	-0.3679	-0.3673	-0.3159	-0.1450
M5	-0.3612	-0.1405	-0.3042	-0.3159	-0.3579
Sum	-1.5218	-1.4444	-1.3876	-1.4602	-1.4427
h_i	0.9455	0.8974	0.8621	0.9073	0.8964
d_i	0.0545	0.1026	0.1379	0.0927	0.1036
w_i	0.1109	0.2088	0.2806	0.1888	0.2109

Step 3

In addition, in this step, the weighted decision matrix, v_{ij} and normalised weighted decision matrix, $v_{ij}{}^*$ are calculated using Equations 7 and 8 respectively. Tables 7 and 8 present the weighted decision matrix, v_{ij} and the normalised weighted decision matrix, $v_{ij}{}^*$, respectively.

Table 7: Weighted decision matrix

Alternative	**P**	**B**	**SS**	**HDC**	**R**
M1	0.0166	0.0569	0.0122	0.0090	0.0201
M2	0.0166	0.0380	0.0244	0.0360	0.0603
M3	0.0333	0.0285	0.0976	0.0719	0.0603
M4	0.0111	0.0759	0.0976	0.0360	0.0100
M5	0.0333	0.0095	0.0488	0.0360	0.0603
Max	0.0333	0.0759	0.0976	0.0719	0.0603

Table 8: Normalised weighted decision matrix

Alternative	**P**	**B**	**SS**	**HDC**	**R**
M1	0.5000	0.7500	0.1250	0.1250	0.3333
M2	0.5000	0.5000	0.2500	0.5000	1.0000
M3	1.0000	0.3750	1.0000	1.0000	1.0000
M4	0.3333	1.0000	1.0000	0.5000	0.1667
M5	1.0000	0.1250	0.5000	0.5000	1.0000

Step 4

In this step, a BACS graph is constructed with their respective weight of attributes. The weight obtained for each attribute are $w_P = 0.1109$, $w_B = 0.2088$, $w_{SS} = 0.2806$, $w_{HDC} = 0.1888$, and $w_R = 0.2109$. The BACS graph with the respective weights is illustrated in Figure 4.

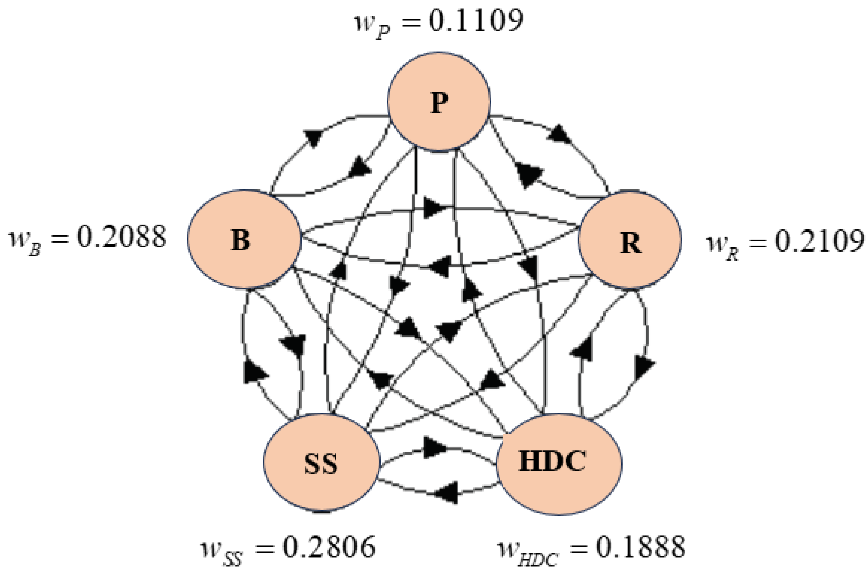

Figure 4: The BACS graph of desktop computer selection

Step 5

Here, the relative importance matrix for each alternative is constructed and shown in Table 9 until Table 13.

Table 9: Pairwise comparison matrix of criteria with respect to M1

	P	**B**	**SS**	**HDC**	**R**
P	0.500	0.750	0.500	0.250	0.500
B	0.250	0.750	0.375	0.750	0.875
SS	0.500	0.625	0.125	0.250	0.250
HDC	0.750	0.250	0.750	0.125	0.250
R	0.500	0.125	0.750	0.750	0.333

Table 10: Pairwise comparison matrix of criteria with respect to M2

	P	**B**	**SS**	**HDC**	**R**
P	0.500	0.750	0.500	0.250	0.500
B	0.250	0.500	0.375	0.750	0.875
SS	0.500	0.625	0.250	0.250	0.250
HDC	0.750	0.250	0.750	0.500	0.250
R	0.500	0.125	0.750	0.750	1.000

Table 11: Pairwise comparison matrix of criteria with respect to M3

	P	**B**	**SS**	**HDC**	**R**
P	1.000	0.750	0.500	0.250	0.500
B	0.250	0.375	0.375	0.750	0.875
SS	0.500	0.625	1.000	0.250	0.250
HDC	0.750	0.250	0.750	1.000	0.250
R	0.500	0.125	0.750	0.750	1.000

Table 12: Pairwise comparison matrix of criteria with respect to M4

	P	**B**	**SS**	**HDC**	**R**
P	0.333	0.750	0.500	0.250	0.500
B	0.250	1.000	0.375	0.750	0.875
SS	0.500	0.625	1.000	0.250	0.250
HDC	0.750	0.250	0.750	0.500	0.250
R	0.500	0.125	0.750	0.750	0.167

Table 13: Pairwise comparison matrix of criteria with respect to M5

	P	**B**	**SS**	**HDC**	**R**
P	1.000	0.750	0.500	0.250	0.500
B	0.250	0.125	0.375	0.750	0.875
SS	0.500	0.625	0.500	0.250	0.250
HDC	0.750	0.250	0.750	0.500	0.250
R	0.500	0.125	0.750	0.750	1.000

Step 6

Here, the permanent function of the matrix for the alternatives is developed and it has $(5+1)$ groups which signifies the existence of attributes and the relative important loops. The number of attributes that exist in the selection is five where the permanent function gives $5!=120$ terms which arranged in $(5+1)$ groups. The permanent of each alternative is represented by mathematical expression in symbolic form as shown below. Consequently, the permanent index value for the permanent function of matrix for each alternative is calculated with the help of MATLAB software.

$$
\begin{aligned}
&\text{Per(B)}\\
&=\prod_{i=1}^{5} R_i + \sum_i \sum_j \sum_k \sum_l \sum_m (R_{ij} R_{ji}) R_k R_l R_m \\
&\quad + \sum_i \sum_j \sum_k \sum_l \sum_m (R_{ij} R_{jk} R_{kl} + R_{ik} R_{kj} R_{ji}) R_l R_m \\
&\quad + \sum_i \sum_j \sum_k \sum_l \sum_m (R_{ij} R_{jk} + R_{kl} R_{lk}) R_m \\
&\quad + \sum_i \sum_j \sum_k \sum_l \sum_m (R_{ij} R_{jk} R_{kl} R_{li} + R_{il} R_{lk} R_{kj} R_{ji}) R_m \\
&\quad + \sum_i \sum_j \sum_k \sum_l \sum_m (R_{km} R_{lm} R_{mk} + R_{km} R_{ml} R_{lk} R_{ji}) R_{ij} R_{ji} \\
&\quad + \sum_i \sum_j \sum_k \sum_l \sum_m (R_{ij} R_{jk} R_{kl} R_{lm} R_{mi} R_{ml} + R_{im} R_{ml} R_{lk} R_{kj} R_{ji})
\end{aligned}
$$

Step 7

The alternatives in the desktop computer selection are ranked according to the associated permanent index value. The alternative with the highest permanent index values is the best option in the selection. The permanent index value for the alternatives in the desktop computer selection is shown in Table 14.

Table 14: The ranking of desktop computer selection

Alternative	BACS with Shannon Entropy	
	Permanent Index	**Rank**
M1	2.9074	5
M2	4.2815	4
M3	8.0396	1
M4	4.5429	3
M5	4.9756	2

According to Table 14, the highest permanent index is 8.0396 represented by alternative M3. This indicates that alternative M3 is the most preferred option n followed by alternative M5. Alternative M4 ranks third followed by alternative M2. Meanwhile, alternative M1 is in last place as it has the lowest permanent index value of 2.9074.

Subsequently, the comparison of ranking results between the proposed research and GTMA and hybrid AHP-TOPSIS are made and given in Table 15.

Table 15: Ranking of desktop computer selection

Alternative	**GTMA**		**BACS with Shannon Entropy**		**AHP-TOPSIS**	
	Permanent Index	**Rank**	**Permanent Index**	**Rank**	**Permanent Index**	**Rank**
M1	1.7392	5	2.9074	5	0.4302	4
M2	2.0056	4	4.2815	4	0.4086	5
M3	2.4750	1	8.0396	1	0.6888	1
M4	2.1026	2	4.5429	3	0.4493	3
M5	2.0997	3	4.9756	2	0.5553	2

Based on Table 15, as for the GTMA, the alternative for desktop computer selection is ranked as M3 with the permanent index of 2.4750, followed by M4 with the permanent index of 2.1026 and M5 with the permanent index of 2.0997. Then, M2 with the permanent index of 2.0056 and lastly M1 with the permanent index of 1.7392.

Meanwhile, for the BACS with Shannon entropy, the alternatives are ranked as M3 with the permanent index of 8.0396, followed by M5 with a permanent index of 4.9756 and M4 with the permanent index of 4.5429. Subsequently, M2 with the permanent index of 4.2815 and lastly M1 with the permanent index of 2.9074. As for the hybrid AHP-TOPSIS, the desktop computer is ranked as M3 with the permanent index of 0.6888, followed by M5 with the permanent index of 0.5553 and M4 with the permanent index of 0.4493.

Then, M1 with the permanent index of 0.4302 and lastly, M2 with the permanent index of 0.4086. Hence, based on the ranking result obtained in Table 14, it shows that the desktop computer of M3 is the preferred choice among the three methods. However, it is observed that slight discrepancies exist in the ranking of intermediate alternatives. These differences may be attributed to the distinct mechanisms employed in each method. In particular, the proposed method incorporates entropy-based objective weighting, which captures the inherent variability of attribute data, whereas GTMA does not explicitly consider attribute weights. Additionally, the transformation of discrete AHP scales into continuous BACS values introduces sensitivity in the evaluation process, which may influence the relative dominance of alternatives. Despite these variations, the high Spearman correlation coefficients (0.8-0.9) as in Table 16 indicate a strong agreement between the methods, thereby validating the robustness and reliability of the proposed approach.

Table 16: Spearman correlation coefficients

Method	**GTMA**	**BACS with Shannon Entropy**	**AHP-TOPSIS**
GTMA	-	0.9	0.8
BACS with Shannon entropy		-	0.9
AHP-TOPSIS			-

Conclusions

This study proposed an integrated MCDM framework by combining the Bounded Autocatalytic Set (BACS) method with Shannon entropy to overcome the limitation of GTMA-based approaches that lack objective of criteria weighting. The incorporation of Shannon entropy enables the determination of attribute weights based on data variability, while BACS captures the interrelationships among attributes. Based on the numerical example, Model M3 obtained the highest permanent index value of 8.0396, indicating that it is the most preferred alternative among the five desktop computer models. The comparison with GTMA and AHP-TOPSIS methods shows Spearman correlation coefficients ranging from 0.8 to 0.9, which indicate strong agreement and confirm the reliability of the proposed method.

The proposed approach successfully addresses the research gap by incorporating objective weighting into the BACS framework, resulting in a more robust and systematic decision-making process. In addition, the method offers advantages such as reduced subjectivity, improved representation of attribute relationships, and better analytical capability for complex decision problems. Despite its effectiveness, this study has some limitations. The numerical example is based on a relatively small dataset with five alternatives and five attributes. Future work may extend this study to larger datasets and explore the use of fuzzy or uncertain data in the decision-making process. The applicability of the proposed method can also be tested in other domains such as supplier selection, healthcare decision-making, and sustainable resource management.

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Conflict of Interest Statement

The authors declare that there is no conflict of interest.

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