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SEA TURTLE-PREDATOR MODEL WITH THE EFFECT OF EXPLOITATION PARAMETER

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ABSTRACT

Sea turtles, vital to marine ecosystems, face considerable threats due to human exploitation, putting their global populations at risk. This study examines a prey-predator model to explore the dynamics of sea turtle populations, focusing on the impact of humans and their interactions with marine predators. The goal is to examine how the dynamics of this model change when the exploitation parameter is altered. Through stability and bifurcation analyses, we assess the conditions that determine whether sea turtle populations remain stable or undergo substantial changes as exploitation pressure fluctuates. We employed eigenvalue analysis to establish the model's stability conditions. Furthermore, bifurcation analysis reveals the system's complex behaviour as the exploitation rate changes. By constructing bifurcation diagrams, we pinpoint critical values at which significant shifts in the dynamics of sea turtle populations occur, including the emergence of alternative stable states or the onset of population decline. The results revealed that once the exploitation rate is below its critical threshold, both the sea turtle as well as predator populations survive and oscillate. Nonetheless, if exploitation is too high, both populations face extinction within 20 to 60 years. Therefore, this research is important for raising awareness about the value of sea turtles and the threats they face, which can help reduce exploitation.

2020 Mathematics Subject Classification:

Introduction

Sea turtles are remarkable marine creatures, yet they have long been subjected to harsh exploitation by humans. This exploitation is driven by economic, cultural, and culinary interests. Despite conservation efforts, sea turtles continue to face significant threats from human activities. To develop effective conservation strategies, it is crucial to understand the scope and impact of human exploitation. Hunting sea turtles has been a widespread cultural practice in most coastal communities globally, aimed at harvesting their meat, eggs, and shells [1, 2].

Several sea turtle species are facing the threat of extinction due to unsustainable nest-site harvesting, driven by the demand for turtle eggs, particularly in Latin America as well as Southeast Asia. Industrial activities such as fishing and coastal development contribute to the exploitation of sea turtles. Accidental capture in fishing gear, particularly in trawl fisheries and longline fisheries, significantly raises annual sea turtle mortality rates. Sea turtle populations are further threatened by habitat degradation as well as destruction caused by climate change, pollution, and the development of coastal infrastructure [3]. Additionally, the request for turtle shell products, valued for their decorative and medicinal properties, has encouraged poaching activities and illegal trade [4]. Studies on sea turtle exploitation have been conducted in various regions around the world, including Mexico [1], India [2, 7], the Mediterranean [5], Madagascar [6], and Malaysia [8-11].

Most of the data on sea turtles collected through sampling have been assessed utilising statistical methods. Nonetheless, fewer studies have applied mathematical models to the sea turtle case. For example, Mazaris and Matsinos [12] developed an Individual-based Model (IBM) to simulate how environmental changes, including density dependence and temporal variability, affecting the population dynamics of sea turtles. They used the green turtle, *Chelonia mydas*, as their model species. Their work resulted in a graph illustrating the likelihood of sea turtle population extinction under distinct scenarios of temporal variability as well as density dependence.

In addition, Roslan *et al*. [13] developed a new dynamic model to study the interaction between sea turtles and humans. They examined the stability of equilibrium points to assess the sustainability of sea turtles. Consequently, their results showed that without human intervention, sea turtles remain in a stable state. The dynamics of humans and sea turtles were illustrated through time series as well as the creation of phase portraits.

Lutfi [14] determined the survival or extinction of the sea turtle population using a discrete model of sea turtle-human interaction, along with the concepts of attractors and basins of attraction from dynamical systems theory. The stability index method, as proposed by Podvigina and Ashwin [15], was applied to this model, revealing that the key factor influencing whether sea turtles survive or become extinct is their initial population size.

Due to the limited references on modelling sea turtles with the effect of exploitation, this paper proposes a straightforward model consisting of two Ordinary Differential Equations (ODEs) for sea turtles and marine predators. Stability analysis, equilibrium points, and bifurcation analysis will be performed on this model, followed by both analytical and numerical examinations of these methods. This study presents a mathematical framework for analysing the impact of conservation efforts, predation, and external mortality on sea turtle populations, providing valuable insights for ecological management and species conservation.

Mathematical Model and Methods

To formulate the mathematical model for the sea turtle-predator with exploitation effect, we consider the following assumptions:

i. Sea turtles grow at a natural rate (r) and conservation rate (α).
ii. They experience mortality due to natural rate (η) and exploitation rate (γ).
iii. Conservation efforts α positively affect sea turtle growth.
iv. The interaction with predators βxy reduces the sea turtle population.
v. Marine predators consume sea turtles at a predation rate βxy.
vi. Predator growth depends on the number of sea turtles consumed, with an efficiency factor e.
vii. Predators experience natural mortality at a rate δy.
viii. The model assumes a closed system with no immigration or emigration of sea turtles and predators.
ix. The predator population depends only on prey availability, with no other resource limitations.

From the above assumptions, the resulting system of ODEs is as follows:

$$\begin{aligned} \frac{dx}{dt} &= \alpha r x - \beta x y - (\eta + \gamma) x\,, \\ \frac{dy}{dt} &= e\beta x y - \delta y, \end{aligned} \tag{1}$$

in which the symbols in the equations above are explained in Table 1. At the same time, the flow diagram in Figure 1 illustrates the schematic representation of the sea turtle and predator model (1). All parameters involved are positive constants.

Table 1: Description of parameters and variables of model (1)

Variable	**Description**
x	Number of sea turtles in the open seawater
y	Number of marine predators
Parameter	**Description**
α	Conservation effort on sea turtles
r	Natural growth rate of the sea turtle
β	Predation rate of sea turtles by natural predators
η	Natural death rate of sea turtle
γ	Exploitation effort on sea turtles
e	Conversion energy rate
δ	Natural death rate of the predator

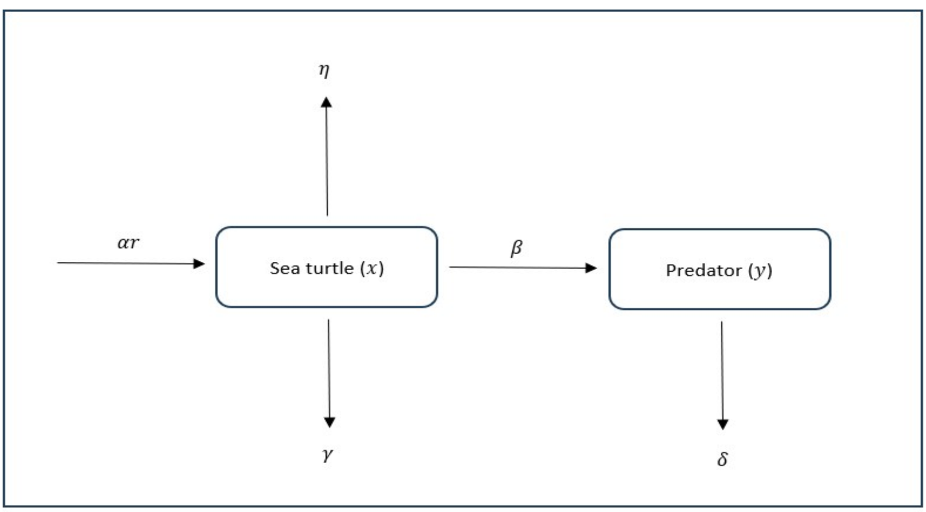


Figure 1: Schematic diagram showing the relationship between marine predators and sea turtles

Equilibria and Stability Theory

In a dynamical system, equilibrium points are conditions in which the system's state variables remain constant over time. Note that an equilibrium point is deemed stable if small disturbances from this point lead to the system returning to equilibrium over time.

Definition 1. (Equilibrium point). An equilibrium point (also known as a fixed point or steady state) refers to a condition where the system's state variables stay constant over time.

Numerically, an equilibrium point is defined by the condition that the derivative (rate of change) of each state variable equals zero. Specifically, for a dynamical system represented by a set of differential equations (DEs):

$$\frac{dx}{dt} = f(x),$$

where x refers to the state variables of the system and $f(x)$ resembles the system's dynamics, an equilibrium point x^* meets:

$$f(x^*) = 0.$$

Each equilibrium point x^* possesses its own stability. The following are the theoretical definitions of stability, as per Glendinning [16].

Definition 2. (Stability theory). An equilibrium point x^* is defined as:

i. Lyapunov stable if and only if for all $\varepsilon > 0$ there exists $\delta > 0$ such that if $|x - y| < \delta$ then $|\varphi(x,t) - \varphi(y,t)| < \varepsilon$.

ii. Asymptotically stable if and only if it is both Lyapunov stable and attractive if $|x - y| < \delta$ then $|\varphi(x,t) - \varphi(y,t)|$ implies to 0 for t approach to ∞.

Theorems 1 and 2 provide the stability conditions given the eigenvalues [17].

Theorem 1. (Classification of stability with real eigenvalues). We say (x^*, y^*) is an equilibrium point of 2-dimensional *ODE* $\dot{x} = f(x,y)$ and $\dot{y} = g(x,y)$ if any of the equivalent conditions are satisfied.

i. If both eigenvalues $\lambda_1 < 0, \lambda_2 < 0$, then (x^*, y^*) is asymptotically stable.

ii. If both eigenvalues $\lambda_1 > 0, \lambda_2 > 0$, then (x^*, y^*) is unstable.

iii. If one of the eigenvalues λ is positive, then (x^*, y^*) is saddle, where saddle implies unstable.

Theorem 2. (Classification of stability with complex eigenvalues). Suppose that $\lambda = a \pm bi$, in which λ represents the complex eigenvalue, $a \in \mathbb{R}, b \in \mathbb{C}$. The stability conditions of (x^*, y^*) are given below:

i. If $a < 0$, then (x^*, y^*) is a stable spiral.

ii. If $a < 0$, then (x^*, y^*) is an unstable spiral.

iii. If $a = 0$, $\lambda = \pm bi$, then (x^*, y^*) is a centre, where a centre is always stable.

The above theorems demonstrate the local stability of equilibrium points. Additionally, the global asymptotic behaviour of solutions can be determined using Lyapunov functions, which generalise potential energy functions [18].

Definition 3. (Lyapunov function). Let (x^*, y^*) be an equilibrium of (1). A function $V: \mathbb{R}^n \to \mathbb{R}$ is known as a Lyapunov function for (x^*, y^*) if for some neighbourhood of W of (x^*, y^*), the given conditions are met:

i. $V(x^*, y^*) = 0$ and $V(x, y) > 0$ for all $(x, y) \neq (x^*, y^*)$ in W, and

ii. $\dot{V}(x^*, y^*) \leq 0$ for all (x^*, y^*) in W.

Bifurcation Theory

We also examine how the stability of the equilibria changes as a system parameter is altered. When the behaviour of a dynamical system changes abruptly with a variation in a parameter, it is considered to have encountered a bifurcation [17]. Instances of bifurcations include transcritical bifurcation, saddle-node bifurcation, period-doubling bifurcation, pitchfork bifurcation, and Hopf bifurcation. Regarding this paper, we employ bifurcation analysis to investigate the impact of specific parameters on the system under study.

Analytical Results

This section presents the analytical results regarding the equilibria, including their positivity, the conditions for the stability of the equilibria, and the identification of the bifurcation point related to the model (1).

Analysis of Equilibria for Model (1)

The equilibrium point refers to where the state variables remain constant and unchanged over time. In most cases, a non-linear system yields multiple equilibrium points. Our study focuses on a non-linear predator-prey model. To determine equilibrium, we resolve the system by setting it equal to zero:

$$\frac{dx}{dt} = \alpha r x - \beta x y - (\eta + \gamma)x = 0, \tag{2}$$

$$\frac{dy}{dt} = e\beta x y - \delta y = 0. \tag{3}$$

Hence,

$$\alpha r x - \beta x y - (\eta + \gamma)x = 0,$$

$$x(\alpha r - \beta y - (\eta + \gamma)) = 0,$$

$$x = 0,$$

and

$$\alpha r - \beta y - \eta - \gamma = 0.$$

Consequently, we rearrange the equation above to obtain:

$$y = \frac{\alpha r - (\eta + \gamma)}{\beta}.$$

To find our first equilibrium point, we substitute $x = 0$ into equation (3), we obtain:

$$0 - \delta y = 0,$$

$$y = 0.$$

Therefore, our first equilibrium point, E_1 is

$$E_1 = (0,0).$$

Correspondingly, we substitute $y = \frac{\alpha r - (\eta + \gamma)}{\beta}$ into Equation (3),

$$\frac{\alpha r - (\eta + \gamma)}{\beta}(e\beta x - \delta) = 0,$$

$$e\beta x - \delta = 0,$$

$$e\beta x = \delta,$$

$$x = \frac{\delta}{e\beta}.$$

Thus, our second equilibrium point, E_2 is

$$E_2 = \left(\frac{\delta}{e\beta}, \frac{\alpha r - (\eta + \gamma)}{\beta}\right).$$

From the E_1 and E_2 above, it is demonstrated that the solution E_1 is non-negative, hence, there is no need to prove its positivity. Nonetheless, we must prove the positivity of the equilibrium point E_2 of when x is obviously a positive value. The proof is accomplished in Lemma 1.

Lemma 1. The solution y in the equilibrium point E_2 is positive if $\alpha r > \eta + \gamma$.

Proof. For y to be positive, we let the solution y be greater than 0:

$$\frac{\alpha r - (\eta + \gamma)}{\beta} > 0.$$

Consequently, we obtain:

$$\alpha r - (\eta + \gamma) > 0.$$

Hence, rearrange to achieve:

$$\alpha r > \eta + \gamma.$$

Therefore, solution y in E_2 is positive if $\alpha r > \eta + \gamma$. This means that the conservation effect, α, together with the natural growth rate, r, must be greater than the values of the natural death rate, δ, and exploitation rate, γ.

Local Stability of Equilibria

In this section, we analyse the stability of the equilibria by determining the eigenvalues with regard to each equilibrium point E_1 and E_2, utilising the Jacobian matrix. The general expression for the Jacobian matrix is given by:

$$J = \begin{bmatrix} \frac{\partial \dot{x}}{\partial x} & \frac{\partial \dot{x}}{\partial y} \\ \frac{\partial \dot{y}}{\partial x} & \frac{\partial \dot{y}}{\partial y} \end{bmatrix}.$$

Thus, the general Jacobian matrix concerning system (1) is

$$J = \begin{bmatrix} \alpha r - \beta y - (\eta + \gamma) & -\beta x \\ e\beta y & e\beta x - \delta \end{bmatrix}.$$

Below, we demonstrate the local stability of the first equilibrium point, which represents the extinction of both predators as well as sea turtles.

Theorem 3. The equilibrium point $E_1(0,0)$ is asymptotically stable if $\alpha r < \eta + \gamma$ and unstable if $\alpha r > \eta + \gamma$.

Proof. Jacobian matrix for E_1 is

$$J_{E_1} = \begin{bmatrix} \alpha r - (\eta + \gamma) & 0 \\ 0 & -\delta \end{bmatrix}.$$

To establish the eigenvalues, we employ $det(J - \lambda I) = 0$

$$det\left(\begin{bmatrix} \alpha r - (\eta + \gamma) & 0 \\ 0 & -\delta \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}\right) = 0.$$

By resolving the determinant

$$\alpha r - (\eta + \gamma) - \lambda = 0.$$

Thus, we achieve

$$\lambda_1 = \alpha r - (\eta + \gamma),$$

and

$$\lambda_2 = -\delta.$$

From the above, it is confirmed that λ_2 is negative. On the other hand, λ_1 may be either negative or positive eigenvalues.

For E_1 to be asymptotically stable, we let $\lambda_1 < 0$, thus

$$\alpha r - (\eta + \gamma) < 0.$$

Rearrange to obtain

$$\alpha r < \eta + \gamma.$$

Thus, E_1 is asymptotically stable if $\alpha r > \eta + \gamma$.

Meanwhile, for E_1 to be unstable, we let $\lambda_1 > 0$

$$\alpha r - (\eta + \gamma) > 0.$$

Rearrange to achieve

$$\alpha r > \eta + \gamma.$$

Thus, E_1 is unstable if $\alpha r > \eta + \gamma$.

Theorem 4. The equilibrium E_2 is stated to be a centre (stable) if $\alpha r > \eta + \gamma$ and unstable if $\alpha r < \eta + \gamma$.

Proof. The Jacobian matrix for E_2 may be written as below:

$$J_{E_2} = \begin{bmatrix} 0 & -\frac{\delta}{e} \\ e(\alpha r - (\eta + \gamma)) & 0 \end{bmatrix}.$$

Considering the determinant

$$det(J_{E_2} - \lambda I) = 0,$$

$$det\left(\begin{bmatrix} 0 & -\frac{\delta}{e} \\ e(\alpha r - (\eta + \gamma)) & 0 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}\right) = 0,$$

$$(-\lambda)(-\lambda) + \left(\delta \cdot \left(\alpha r - (\eta + \gamma)\right)\right) = 0.$$

Hence, we get

$$\lambda_1 = \sqrt{\delta\eta + \delta\gamma - \alpha\delta r},$$

and

$$\lambda_2 = -\sqrt{\delta\eta + \delta\gamma - \alpha\delta r}.$$

From the above, it is confirmed that λ_2 is negative. We predict that E_2 may be a centre or an unstable equilibrium point. E_2 is a centre if

$$\delta\eta + \delta\gamma < \alpha\delta r.$$

Therefore,

$$\eta + \gamma < \alpha r,$$

or $\alpha r > \eta + \gamma$. Hence, both eigenvalues become

$$\lambda_{1,2} = \pm\sqrt{\alpha\delta r - \delta\eta - \delta\gamma}\,i,$$

having pure imaginary complex eigenvalues. For E_2 to be unstable, we assume $\lambda_1 > 0$. Then, one has that

$$\sqrt{\delta(\eta + r - \alpha r)} > 0,$$

$$\left(\sqrt{\delta(\eta + r - \alpha r)}\right)^2 > 0,$$

$$\delta(\eta + r - \alpha r) > 0,$$

$$\eta + r - \alpha r > 0,$$

$$\alpha r < \eta + \gamma.$$

Thus, we may conclude that E_2 is unstable if $\alpha r < \eta + \gamma$.

Global Stability Analysis for Coexistence Equilibrium Point $E_2(x^*, y^*)$

In this section, we employ the Lyapunov function to analyse the global stability behaviour of the coexisting equilibrium E_2. A potential choice for the system (1) is a quadratic Lyapunov function

$$V(x, y) = \frac{1}{2}(x^2 + y^2).$$

The derivative concerning $V(x, y)$ along the trajectories of the system may be measured as

$$\frac{dV}{dt} = \frac{\partial V}{\partial x}\frac{dx}{dt} + \frac{\partial V}{\partial y}\frac{dy}{dt}.$$

Thus,

$$\frac{dV}{dt} = (x)(\alpha r x - \beta x y - \eta x - \gamma x) + (y)(e\beta x y - \delta y),$$

$$= x^2(\alpha r - \beta y - \eta - \gamma) + y^2(e\beta x - \delta),$$

$$= -x^2(\beta y + \eta + \gamma - \alpha r) - y^2(\delta - e\beta x).$$

Therefore, $V(x, y)$ is a Lyapunov function given that $(\beta y + \eta + \gamma - \alpha r)$ and $(\delta - e\beta x)$ are positive regarding some neighbourhood of the coexisting equilibrium $E_2(x^*, y^*)$. Having these conditions, we may conclude that the equilibrium E_2 is globally asymptotically stable.

Bifurcation Point Formula for Model (1)

In this study, we aim to examine the changes in stability by varying the exploitation parameter γ in model (1). Therefore, we need to determine the critical value regarding the parameter that results in purely imaginary eigenvalues. We proceed to solve for the parameter γ as below:

From the first eigenvalue of E_1, we obtained

$$\lambda_1 = \alpha r - (\eta + \gamma).$$

We let $\lambda_1 = 0$ so that

$$\alpha r - (\eta + \gamma) = 0,$$

$$\alpha r = (\eta + \gamma).$$

Solve for $\gamma = \gamma_c$,

$$\gamma_c = \alpha r - \eta.$$

This indicates that the critical value of the exploitation parameter γ is given by $\alpha r - \eta$. Thus, we will track the types of stability for $\gamma > \alpha r - \eta$ or $\gamma < \alpha r - \eta$ in the following section.

Numerical Results

In this section, we conduct a numerical simulation of model (1) utilising Maple software.

Stability of Equilibrium Point of Model (1)

Recall that, for model (1), we identified two equilibrium points, which are $E_1 = (0,0)$ and $E_2 = (\frac{\delta}{e\beta}, \frac{\alpha r-(\eta+\gamma)}{\beta})$. The set of parameter values employed is referred to in [11] which are $r = 1.3, \alpha = 0.92,\ \beta = 0.01, \eta = 0.25, \gamma = 0.08, e = 1, \delta = 0.25$. The general Jacobian matrix of model (1) is given by:

$$J = \begin{bmatrix} 0.866 - 0.01y & -0.01x \\ 0.01y & 0.01x - 0.25 \end{bmatrix}.$$

The eigenvalues concerning each equilibrium point are presented in Table 2.

Table 2: Eigenvalues and types of stability for equilibria in model (1) for various values of exploitation effort concerning sea turtles

Exploitation Effort on Sea Turtles, γ	**Equilibrium Point**	**Eigenvalues**	**Stability**
$\gamma = 0.08$	$E_1 = (0,0)$	$\lambda_1 = 0.866$ $\lambda_2 = -0.250$	Unstable
	$E_2 = (25,86.6)$	$\lambda_1 = 0.465i$ $\lambda_2 = -0.465i$	Centre
$\gamma = 0.2$	$E_1 = (0,0)$	$\lambda_1 = 0.746$ $\lambda_2 = -0.250$	Unstable
	$E_2 = (25,74.6)$	$\lambda_1 = 0.432i$ $\lambda_2 = -0.432i$	Centre
$\gamma = 0.6$	$E_1 = (0,0)$	$\lambda_1 = 0.346$ $\lambda_2 = -0.250$	Unstable
	$E_2 = (25,34.6)$	$\lambda_1 = 0.294i$ $\lambda_2 = -0.294i$	Centre

$\gamma = 1$	$E_1 = (0,0)$	$\lambda_1 = -0.054$ $\lambda_2 = -0.250$	Asymptotically stable
	$E_2 = (25, -5.4)$	$\lambda_1 = 0.116$ $\lambda_2 = -0.116$	Unstable
$\gamma = 2$	$E_1 = (0,0)$	$\lambda_1 = -0.250$ $\lambda_2 = -1.054$	Asymptotically stable
	$E_2 = (25, -105.4)$	$\lambda_1 = 0.513$ $\lambda_2 = -0.513$	Unstable

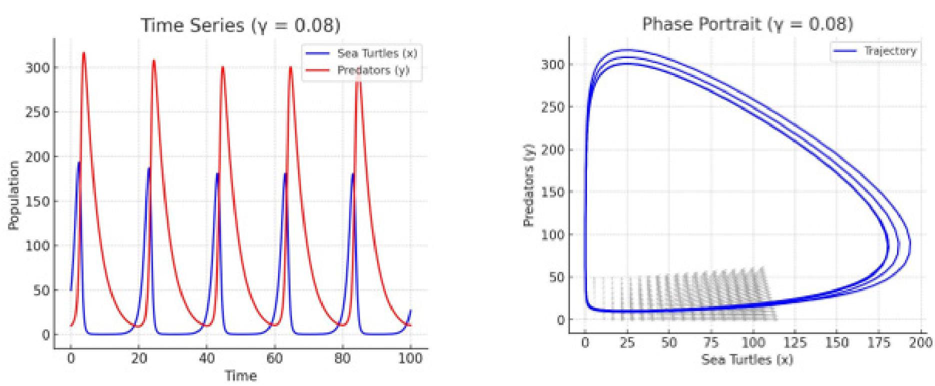
These findings vary with distinct values of exploitation effort regarding sea turtles, γ. By considering γ = 0.08, 0.2, 0.6, the equilibrium E_1 possesses one positive eigenvalue, indicating that E_1 is unstable by Theorem 1. Meanwhile, E_2 has complex conjugate eigenvalues having no real parts. Hence, by Theorem 2, E_2 represents a centre. When an equilibrium point is a centre, it indicates that the system's dynamics lead to oscillations around that point, without spiralling outward or converging toward it. Instead, the system continuously orbits the equilibrium point, sustaining a constant distance while oscillating. However, it is noted that a centre is always stable, but not asymptotically stable.

Then, by considering γ = 1, 2, the equilibrium E_2 is excluded as it holds no biological relevance. A scenario in which one or more populations have negative values is referred to as a negative equilibrium point. Meanwhile, the only equilibrium left is E_1, where E_1 is asymptotically stable. From Table 2, we may also demonstrate that E_1 is changed from unstable to asymptotically stable, whilst E_2 changed from centre to unstable, as γ increases. In the bifurcation section, we will present the critical value of γ at which these equilibria switch their stability. The following section will simulate the phase portrait and time series plot for the same values of γ.

Time Series and Phase Portrait

In this section, time series as well as two-dimensional phase portraits are generated for various values of the exploitation effort concerning sea turtles, γ, as shown in Figure 2. Note that the time series plotted in these figures employed the same initial condition ($x(0)$, $y(0)$) = (10, 10). Maple software was utilised to generate the plots.

a)



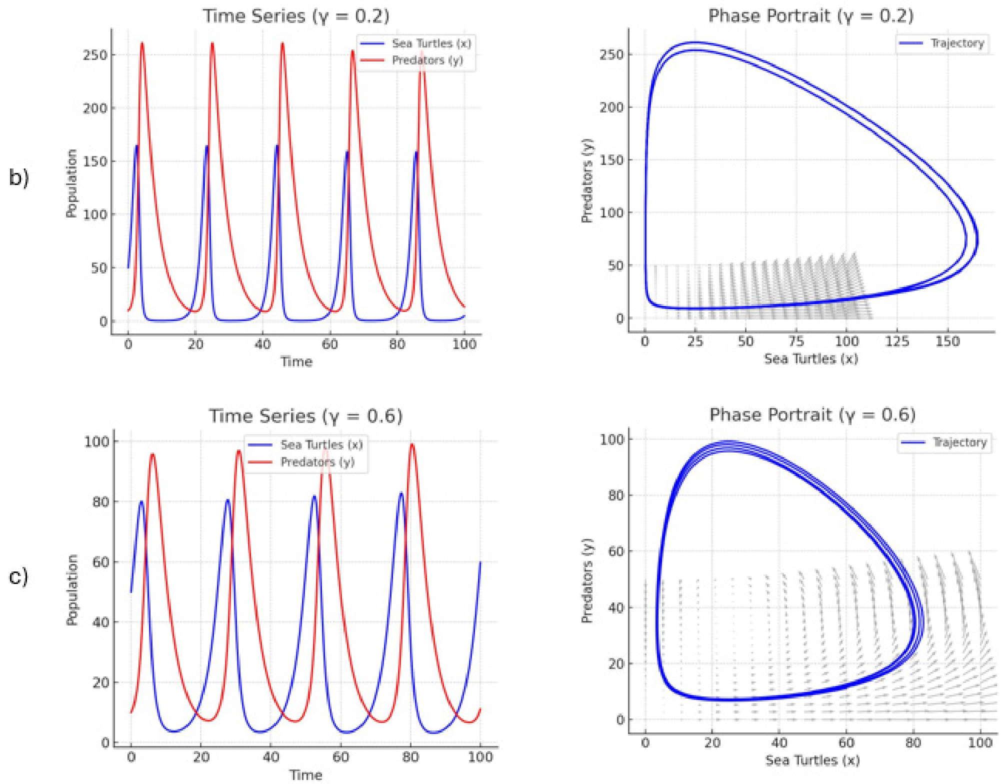


Figure 2: Time series and phase portrait for (a) $\gamma = 0.08$, (b) $\gamma = 0.2$, (c) $\gamma = 0.6$

When γ = 0.08, 0.2, 0.6, the population of sea turtles in marine predators and open water is increasing and oscillating, as depicted in Figure 2. Low exploitation efforts have little impact on the populations of both sea turtles and predators. The phase portrait on the right of Figure 2 illustrates that the equilibrium point E_2 = (25,86.6), (25,74.6), (25,34.6), indicates a centre pattern, suggesting that the system is stable and the sea turtle population will be maintained in the future.

Nonetheless, with high exploitation efforts on sea turtles, γ = 1, 2, the populations of sea turtles as well as predators are predicted to go extinct in about 80 and 20 years, respectively (Figure 3). This is due to the exploitation efforts that have caused the extinction of both populations. These findings highlight the crucial impact of exploitation effort in model (1). Therefore, comprehending the dynamics of model (1) at various levels of this factor is crucial for effective ecosystem management and conservation.

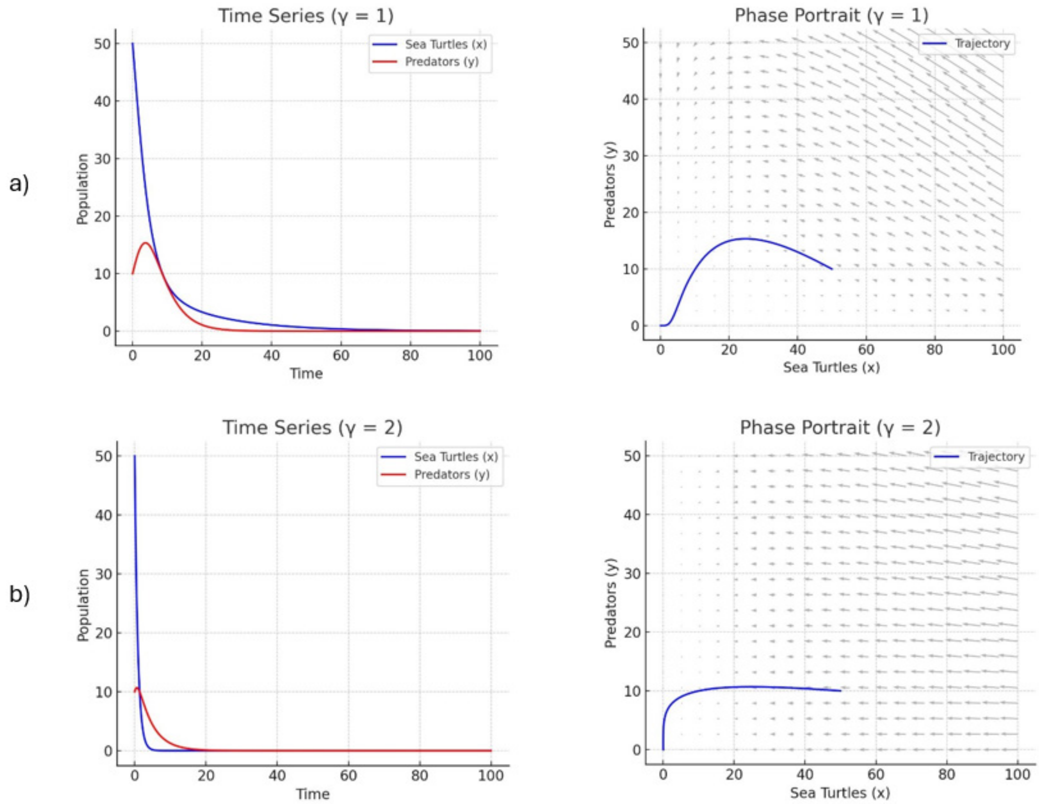


Figure 3: Time series and phase portrait for (a) $\gamma = 1$ and (b) $\gamma = 2$

Bifurcation Analysis for Exploitation Parameter

We selected the exploitation parameter γ as the bifurcation parameter. Based on the analytical results in the previous section, the formula for the critical value of γ is given by:

$$\gamma^* = \alpha r - \eta.$$

By substituting values of $\alpha = 0.92, r = 1.3$, and $\eta = 0.25$, we obtained that the bifurcation point occurred at $\gamma^* = 0.946$. From Table 2, the equilibrium E_1 changes from unstable to stable, while E_2 moves from stable to unstable. The stability changes after crossing the bifurcation point $\gamma^* = 0.946$. Thus, this supports that model (1) encountered a transcritical bifurcation.

Figures 4 and 5 show the bifurcation diagrams for sea turtles (x) as well as marine predators (y) in relation to the exploitation parameter. The considered range is for $\gamma \in [0,2]$, as it is assumed that the highest exploitation rate is 2. $\gamma = 2$ indicates that the number of sea turtles exploited each year is twice that of the previous year, due to increased human activity.

In Figure 4, $E_2(x^*, y^*)$ is stable for the first region $0 \leq \gamma \leq \gamma^*$ and for the second region $\gamma^* > \gamma \geq 2, E_1$ is stable. This suggests that at low exploitation rates, the population remains stable at $x = 25$, but abruptly becomes extinct when the exploitation rate is high. A different scenario is observed in Figure 5, where the population concerning marine predators decreases linearly in the first region as the exploitation parameter increases, eventually becoming extinct once it reaches the second region. These findings indicate that the exploitation rate has a greater impact on predators than on sea turtles. This is because harming sea turtles will not directly reduce such populations, as there is another factor that supports the sea turtles' growth, namely conservation efforts.

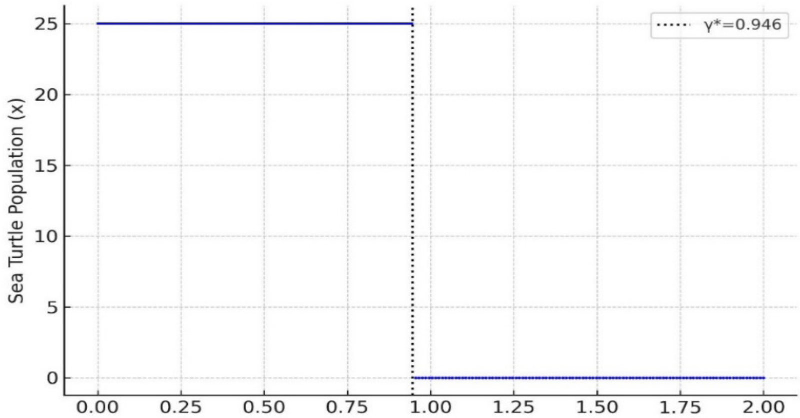


Figure 4: Bifurcation diagram for the population of sea turtles versus parameter γ

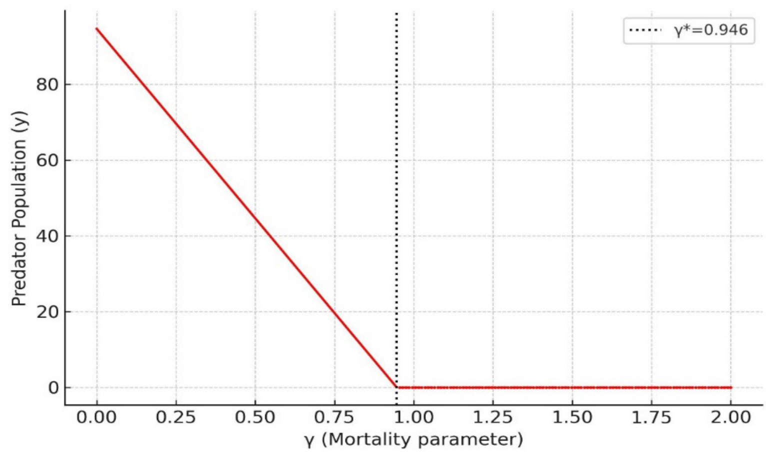


Figure 5: Bifurcation diagram for the population of marine predator versus parameter γ

Conclusions

To conclude, we have effectively analysed a prey-predator model involving sea turtles and marine predators, with the exploitation rate as the key parameter. We identified two equilibrium points in the model: the extinction of both populations and their coexistence. For the latter equilibrium point, positivity conditions were established, making it biologically meaningful. Stability analysis was then performed on these equilibria. Our findings showed that both populations survive and oscillate at low exploitation rates. However, due to excessive exploitation, both populations became extinct, with their disappearance occurring within 20 to 60 years, as recorded in historical documents.

Furthermore, we were able to identify the critical value regarding the exploitation parameter at which the stability of both equilibria shifts by applying the bifurcation concept. This critical value is $y^* = 0.946$. This merely suggests that humans have exploited approximately 96.4% ≈ (or 95%) of sea turtles. A survey by Poti *et al*. [10] on villagers from Redang Island revealed that 86% of them had not participated in any awareness programmes about sea turtle conservation. Therefore, increasing public awareness about the importance of sea turtles and the threats they face could help reduce exploitation. This awareness can be promoted through media campaigns, community outreach, and educational programmes in schools.

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Conflict of Interest Statement

The author declares no conflict of interest.

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