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EXPLORING AI WITH SENSE THROUGH APPLYING THE GRAVITY IN MIND MECHANISM

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ABSTRACT

The study of the laws of motion has been advancing, with significant contributions from key figures like Galileo and Newton. Analogous to the gravitational forces observed in the natural world, individuals occasionally find themselves irresistibly drawn to specific entities. The gravity in mind, the basis of free-fall motion in one's mind, acts as a sensor to make an individual sense subtle judgments about things like common sense, as if it were whispering to our minds. Since it has been said for more than half a century that judging common sense is the most difficult task for AI, this paper explores whether AI can possess true intelligence by applying this mechanism. Empirical data from many different types of games show that Game Refinement (*GR*) zone is located in 0.07-0.08, which respectively corresponds to the lower limit (fairness) and upper limit (engagement). In other words, there is a border between objectivity and subjectivity in a thing, and this is the minimal objectivity, or the resignation in game context. Based upon this, in unconventional circumstances, when a greater gravitational acceleration operates within the mind, a sense of "playfulness" is generated, disrupting the harmony of comfort and discomfort sustained by the gravity in mind. The study concludes that applying the "gravity in mind" mechanism to AI could significantly blur the line between human and artificial intelligence, enhancing AI decision-making capabilities.

2020 Mathematics Subject Classification:
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Introduction

The question of whether AI possesses true intelligence, or wisdom has long been a controversial and extensively discussed topic in the field of Artificial Intelligence (AI). Currently, there is no universally accepted definition for determining what constitutes true intelligence. However, one mainstream view, functionalism, posits that if a system can exhibit behaviours and capabilities similar to human intelligence, then it can be considered intelligent, regardless of its internal implementation. The Turing Test falls within this category [1]. Another mainstream perspective, biologism argues that only biological neural systems akin to the human brain can produce true intelligence, as they possess consciousness. Nonetheless, current AI systems have demonstrated certain intelligent characteristics, but whether they can achieve *artificial general intelligence* comparable to human intelligence remains an extremely challenging research focus in this field.

To date, no research paper has analysed this issue from the perspective of the gravity in mind which contends that the mind uses physical gravity as a mental model or simulacrum to express the relation between the inner self and the outer world [2]. This paper aims to explore whether AI can learn to possess true intelligence by applying the gravity in mind mechanism. To begin with, gravity in mind, is based on the theoretical foundation laid down by the game refinement theory [3]. At the core of the theory is the notion of the *GR* zone value, which implies that there is a link between fairness and competition in games. To be specific, the basic notion of game playing depends on the rate of information representation, which constitutes the velocity of a game, is generally defined as the success rate v. In the game refinement theory, the second derivative (called *GR* value) corresponds to the acceleration in the sense of dynamics. The changes involved in acceleration are viewed as jerk j, which plays a crucial role in motions and can cause discomfort, relating to the retention of motivation [4,5]. With the (v, a, j) parameters utilized in the following three equations: (1) , (2) with and (3) with , we can obtain three cross points, which respectively represents the bound for effort, achievement, and discomfort [4]. Its first (acceleration a) and second (jerk j) derivatives were associated with speculative psychology, such as engagement and unpredictability, a jerk in games is interchangeably regarded as the association of surprise, which is denoted as *AD* value [5]. These concepts are extended to the realm of thought, drawing analogies between physical motion and mind motion. By understanding the motion in the mind through the "gravity in mind" analogy, this paper seeks to bridge the gap between physical performance and mental experience, offering new insights into AI's potential for true intelligence. In essence, these two theories proposed in sequence implies the process of solving uncertainty of the game outcome is as if it were a free-fall motion (uniformly accelerated motion).

A New Perspective: From Universal Gravitation to Gravity in Mind

The study of free-fall motion has captivated scholars and scientists for centuries, leading to ground-breaking discoveries and insights that have shaped our understanding of the fundamental laws of physics. This body of research has explored various aspects of free-fall, including the behaviour of objects in motion, the effects of air resistance, and the underlying principles of gravity and inertia. In this section, this paper will first examine some of the most significant contributions and experiments that have advanced the current knowledge of free-fall motion. Then, this paper will propose a paradigm shift from the motion of object to the motion of thing based on the gravity in mind principle. The gravity in mind principle has many potential applications, including the use as a sensor to judge the pros and cons of a given thing. Moreover, we focus on the jerk, the change of acceleration, which plays a crucial role in motions and can cause the experience of discomfort, relating to the retention of motivation.

Early Works on Free-Fall Motion with Universal Gravitation

One of the earliest and most influential figures in the study of free-fall motion was Galileo Galilei (1564-1642). Through a combination of thought experiments and practical demonstrations, Galileo challenged the prevailing Aristotelian notions of motion and laid the foundations for our modern understanding of gravity and inertia. In his seminal work entitled "Dialogues Concerning Two New Sciences" (1638), Galileo [6] described his famous thought experiment involving objects of different masses being dropped from the Leaning Tower of Pisa. He posited that, in the absence of air resistance, all objects would fall at the same rate, regardless of their weight or composition. This directly contradicted the Aristotelian belief that heavier objects should fall faster than lighter ones. To study falling bodies more systematically, Galileo devised an ingenious experiment using inclined planes and rolling balls. By carefully timing the descent of balls down gently sloping grooves, he could minimise the effects of air resistance and friction. His measurements revealed that the distances travelled by the balls were proportional to the squares of the elapsed times – a pivotal discovery that pointed to the existence of uniform acceleration in free-fall motion. Through meticulous observations and mathematical analysis, Galileo deduced that falling bodies undergo a constant acceleration, which he termed "the acceleration of gravity" or "the natural acceleration." He recognised that, in a vacuum, all objects fall with the same constant acceleration. This acceleration, conventionally denoted as *g*, was quantified as approximately near the Earth's surface. In other words, Galileo's theory of gravitational acceleration applies to all objects on Earth, including humans. According to Galileo's theory, all objects experience the same gravitational acceleration at the surface of the Earth, which is approximately . This means that whether it is a person or any other object, in the absence of air resistance, they will all fall freely with this constant acceleration.

Building upon Galileo's ground-breaking work, Sir Isaac Newton [7] made seminal contributions to the study of free-fall motion through his famous laws of motion and the theory of universal gravitation. Newton's first law, also known as the principle of inertia, established that an object at rest remains at rest, and an object in motion continues to move at a constant velocity unless acted upon by an external force. His second law defined the relationship between an object's mass, the applied force, and the resulting acceleration, providing a quantitative framework for understanding the motion of objects under the influence of forces, including gravity. This law laid the foundation for the mathematical description of free-fall motion and the calculation of gravitational acceleration. Newton's third law, the principle of action and reaction, further solidified the understanding of how forces interact between objects, laying the groundwork for the study of gravitational interactions.

In 1687, Newton [7] published a ground-breaking book called Mathematical Principles of Natural Philosophy, describing gravitation as a universal force, and claimed that "the forces which keep the planets in their orbs must be reciprocally as the squares of their distances from the centres about which they revolve." Perhaps Newton's most significant contribution was the formulation of the universal law of gravitation, which described the attractive force between any two masses in the universe. This law not only explained the motion of objects in free-fall but also provided a unifying framework for understanding the motions of celestial bodies, revolutionising our comprehension of the cosmos.

A Paradigm Shift from the Motion of Object to the Motion of Thing

Galileo [6] contemplated the laws of falling bodies, suggesting that the motion of a falling object is independent of its mass and that the distance of fall is proportional to the square of time. Newton [7] attempted to formalise the motion of free fall based on the concept of universal gravitation. However, Einstein later focused on the gravitational acceleration underlying free fall, proclaiming that gravity is the curvature of space-time [8].

While Hegel and Engels, through the lenses of dialectics and materialism, philosophically interpret the laws of motion of objects [9]. Hegel regards motion as a form of world development, emphasizing the inherent contradictions and unity in the development of things through dialectical thought. Therefore, the understanding of the laws of motion evolves and develops within the dialectical process. On the other hand, Engels emphasises that the motion and changes in the material world stem from internal contradictions and struggles. He views the laws of motion in the natural world as expressions of the development of the material world, arising from the interactions and forces between objects. For the perspective of the motion in mind [2], a profound examination of "what is freedom" and the proclamation that "freedom is insight into necessity" can be seen as laying the foundation for discussing the laws of motion.

To understand this, it's necessary to review the law of free-fall motion. When there is no external force acting on a freely falling object, it will undergo motion characterised by constant acceleration. This means that the object's velocity in the vertical direction will continuously increase, with the rate of increase in velocity remaining constant, i.e., the acceleration remains constant. According to the laws of classical mechanics governing free fall, the trajectory of the object will be a straight line vertically downward, with velocity increasing linearly with time, and the position changing quadratically with time. However, the introduction of external forces leads to a loss of predictability in the trajectory of the motion. Free-fall motion, governed solely by the force of gravity, follows a predictable path due to the constant acceleration experienced by the object. Yet, with the involvement of external forces such as air resistance or other impeding factors, the motion becomes subject to additional complexities. Air resistance, for instance, increases with velocity, altering the object's acceleration and thus disrupting the previously deterministic trajectory. Consequently, the presence of external forces necessitates a more intricate analysis, as it introduces variables that render the trajectory of free-fall motion unpredictable.

Drawing upon the gravity in mind theory, it could be considered that outcomes of all things are inevitably realised. In many cases, the outcomes are not strictly predetermined, implying that there exist multiple potentialities for the outcomes of things. In essence, there is a degree of freedom inherent in things, and it is the potential freedom that constitutes the essence of those things. Therefore, the validity of characterising things based on their degrees of freedom is ensured. If we consider things as a series of decision-making events, strategic games such as Go and Chess serve as typical examples. Similar to today's AI, whose capabilities far surpass those of masters, while there are numerous options available at each juncture, the choice of move is ultimately confined to the optimal one, resulting in a scenario where "freedom equals necessity" or "freedom leads to the inevitable" is realised. The essence of things lies in their degree of freedom, and the process leading to the outcome of things can be understood as insight into necessity.

Considering that human beings are often deeply attracted by certain things, sometimes becoming so addictive that they forget themselves. This phenomenon of being strongly attracted to certain things may bear resemblance to the sensation of objects being pulled towards the Earth's surface. Therefore, the process of recognising the outcome of things over time is analogous of free fall motion in the realm of thought.

The Gravity in Mind as a Sensor to Judge the Common Sense

In general, competitive events such as games proceed in a state of uncertainty, where the outcome becomes evident at a certain point in time. Often, this occurs during what can be referred to as the final highlight, where there is a sudden transition from uncertainty to certainty that causes one to startle. This sensation is akin to the feeling experienced during the rapid descent of a roller coaster

[10], where there is a sudden change that elicits a startled response. This suggests the existence of something akin to gravitational acceleration in the realm of thought— a "something" that influences human cognition akin to a sensor. This is what we refer to as "gravity in mind," a presence capable of influencing human cognition.

An illustration presenting the location of sophisticated board games is given in Figure 1. Each game is located at the cross-point between and , where and *GR* is in the border of the zone values (0.07 and 0.08). Chess and Go is located close on the line and , respectively. Meanwhile, Shogi is located at the intermediate between Chess and Go.

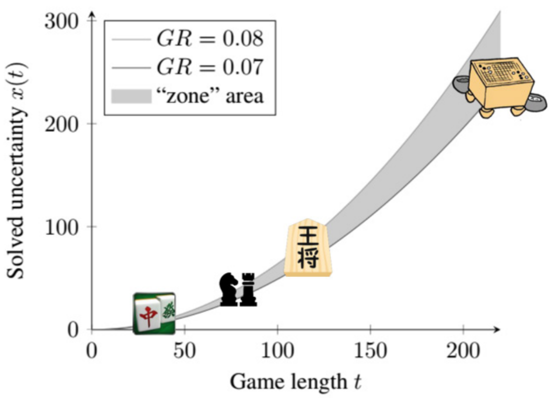

Figure 1: Sophisticated board games such as Chess, Shogi, and Go stand at the cross-points between and where and *GR* is a zone value. Each sophisticated board game is nearly located on the line where chess and shogi being beneath the line and Go being beyond the line.

There is a notion of timing in all things, and once the opportune moment has passed, it becomes redundant and uninteresting. Miscalculating the opportune moment may expose not only intellectual weakness but also, at times, reveal immaturity. In such cases, redundancy is no longer acceptable. This can lead to situations where one may inadvertently harm others or engage in behaviour that deviates from common sense. Common sense, a phrase that human beings are very familiar with, which refers to the basic level of practical knowledge and sound judgment that is widely shared by most people in a given community or culture. It can be thought of as the collective wisdom and intuitive understanding that guides everyday behaviour and decision-making. Since it is a shared body of knowledge and beliefs that is widely accepted and understood within a particular community or culture, "it can't be reduced to zeros and ones." In other words, "a person will search for a different approach or cunningly change the subject. The trouble with computers today is they're always starting from scratch" [11].

The ability to discern the opportune moment, or "gravity in mind," transcends specific domains and possesses a universal character. For instance, within competitive contexts such as games, the faculty of sensing the opportune moment is manifested in the appropriate duration of the match. Games involve a temporal transition from an uncertain state of outcomes to a definitive state, where there exists a degree of freedom for any eventuality during the uncertain phase. However, in strategic games where each participant strives for optimal outcomes, choices become deterministic and inevitable. In situations characterized by uncertain outcomes, judgments of superiority or inferiority remain subjective, whereas in situations of determined outcomes, such judgments become objective. Consequently, the temporal progression of match outcomes signifies a shift from contingency (freedom) to necessity and a transition from subjectivity to objectivity. Ultimately, the confluence of subjectivity and objectivity marks the opportune moment. The "gravity in mind" serves as a sensory mechanism not only for discerning the opportune moment of game conclusion but also for fulfilling a role akin to a sensor that engages with judgments of common sense and nuances of the psyche.

In traditional strategic games like Shogi, there exists a customary practice known as "resignation" which signifies conceding defeat. Resignation is an act of acknowledging defeat, and it is customary to resign when the possibility of a comeback is deemed improbable. As the moment of resignation approaches, both players instinctively explore the timing of resignation as part of their strategic manoeuvring. Resignation allows for the maximisation of the lingering resonance of the game play without diminishing the artistic value inherent in the game records. Therefore, in traditional strategic games, athletes at the outset of the game transform into artists by its conclusion. All these transformations are made possible by the operation of the sensor known as "gravity in mind."

The Gravity in Mind Brings "Charm of the Unnatural"

Under conditions of gravity that deviate from the norm (), individuals may experience sensations of discomfort or exhilarating thrills, which are not typically encountered in everyday life. Examples of such unconventional events may include sudden accelerations in a car or riding on a roller coaster. At the core of these experiences lie variations in gravitational acceleration or jerks as a derivative pf acceleration. Assuming that the gravitational force of the mind corresponds to normal gravity. In unconventional situations, that is, when a greater gravitational acceleration operates in the world of thought, a sense of "playfulness" is generated. Therefore, in contrast to the gravitational force of the mind, we shall refer to this as "gravity of play." In the environment of gravity of play, individuals may experience an unconventional sense of unnaturalness compared to the everyday comfort and naturalness induced by the regular gravitational force of the mind.

Any given phenomenon can be considered as a fusion of the gravitational force of the mind (a quadratic function) and jerk (a cubic function). On the contrary, novel phenomena can be generated by appropriately integrating the gravity in mind and jerk in mind according to the purpose. Gravity in mind imparts naturalness and comfort to the human mind, akin to its natural state and comfort is experienced when navigating within the usual gravitational space. However, in a gravitational environment different from the gravitational force of the mind (gravity of play), a non-routine state is accentuated, leading to factors such as unnaturalness, discomfort, or dissonance. This creates an impression akin to being in a space different from the usual gravitational space. Nonetheless, harmonising naturalness and unnaturalness in moderation deepens comfort. Profound artistic works and captivating phenomena serve as typical examples of this. Iida refers to this as the "charm of the unnatural."

Revisiting the game progress model [2], let t be the game length, and $y(t)$ be the function of the game progress's solved uncertainty, then the ratio of solved uncertainty can be briefly described as . Considering the physic kinematic equation on motions (+) where at , then was obtained. Figure 2 showed that sophisticated board games (Chess, Shogi, and Go) were located on the same curve of

. Since corresponds to the in-game acceleration, the figure depicted that sophisticated games have almost the same "gravity." This condition implies those games to be attractive to play. However, what makes the game different? Interestingly, the jerk (or vibration) of the game was observed to be different. Typically, vibration is, by its nature, a factor of noise or discomfort. However, once a player becomes familiar with such a vibration effect, it would be indispensable (greater engagement) or addictive like a drug while feeling stronger than comfort.

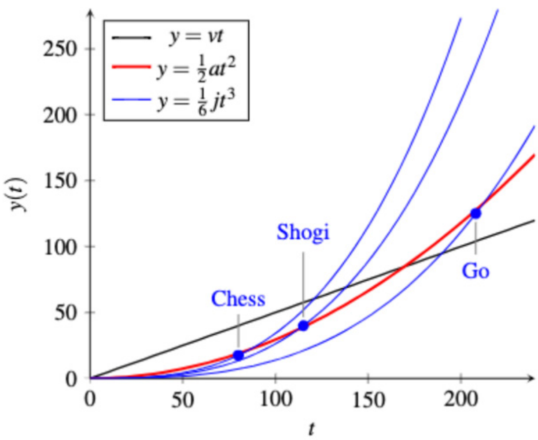

Figure 2: Acceleration and jerk in games (Iida & Khalid [2])

Domain Expertise + (Common) Sense = True Intelligence

The strength of AI in strategic games as well as its remarkable performance in various fields are well known. However, due to the lack of the ability to perceive the tide, truly intelligent AI has not yet been achieved. For example, in a match between a professional shogi player and an AI, if the AI loses, it is truly embarrassing, but it will continue the match endlessly even if there is no chance of a comeback. This only exposes the decline in knowledge and the clumsiness. In a general context, it would be equivalent to causing harm to others without hesitation. Although AI possesses strength far beyond that of humans, it is truly perplexing, because "computers can only do logical things, and meanings are not always logical" [11]. When AI acquires the gravity in mind, it will serve as a sensor and be able to resign at appropriate moments. In an evenly matched competition, it would never lose to humans, but it could cooperate in shaping the outcome as a winner.

Resignation and shaping in traditional strategic games are forms of co-creation with the opponent player, demonstrating empathy and compassion towards the loser (others). It is not about strength or intelligence, but about acquiring true wisdom. In other words, the mere pursuit of "strength" is inadequate. If artistic endeavours could be categorised into temporal arts and spatial arts, then activities such as strategic games, music, and literature would fall under the domain of temporal arts. However, without possessing a sufficient degree of sense, these temporal arts become unworthy of appreciation. In traditional games, such as Shogi, this sense of character manifests itself in the graceful manner of resignation. Temporal arts possess the ability to profoundly touch the human soul, evoking powerful emotions. From a physical standpoint, such an evocation of emotion necessitates the presence of an applied force, which, by its very nature, implies the existence of acceleration. In essence, "domain expertise + (common) sense = true intelligence." At the core of this lies the judgment of common sense based on context and relationships. If the gravity in mind behaves as a sensor of common sense, then in terms of intelligence, there may be no distinction between humans and AI, or perhaps due to the subtlety of its knowledge, it may become an object of respect in the surrounding environment.

Summary

By applying the "gravity in mind" theory to explore AI processes, this research paper has made some insightful findings and discovered alternative perspectives. The results and related discussions are summarised in the following section.

Game Data Analysis Results

This study analysed empirical data from various types of games, studying the patterns of uncertainty, engagement, and *GR* zone values. The results showed: the most highly entertaining and engaging games had *GR* values ranging between 0.07 and 0.08. This range seems to reflect a critical point where games maintain an appropriate balance of fairness and competitiveness, sustaining players' interest and investment.

Different game types exhibit significant differences in their "jerk" values. Generally, higher jerk values indicate stronger uncertainty and surprise in the game, but excessively high values may lead to discomfort and aversion from participants. These findings support the potential application of the "gravity in mind" theory in evaluating game experiences and judging an "optimal moment." The progression of games seems to be influenced by laws of motion similar to those governing free-fall.

The Connection between Free-fall Motion and Motion in Mind

Galileo demonstrated through various experiments that free fall motion in the physical world exhibits a constant and unchanging gravitational acceleration [6]. Similarly, Iida et. al (2004) calculated the *GR* values for different games, all falling within the range of between 0.07 and 0.08. This may be regarded as a constant acceleration akin to "motion in mind." When the gravitational acceleration is at a normal level (similar to human "gravity in mind"), AI exhibits good judgment, able to grasp opportune moments and make humane decisions (such as "resigning"). As gravitational acceleration increases (corresponding to a "gravity of play" environment), AI behaviour starts to exhibit unnatural, erratic and inappropriate characteristics, such as prolonging hopeless matches or obsessing over "winning or losing."

These findings suggest that the influence of "gravity in mind" on AI's judgment ability is similar to its effect on humans. An appropriate level of "gravity" allows AI to maintain a calm, natural, and reasonable cognitive judgment process, while deviations in "gravity" may diminish its ability to exhibit human-like qualities.

The results of this study indicate that the "gravity in mind" mechanism may be the key for AI systems to acquire human-like intelligence and common-sense judgment. If AI could possess a "gravity in mind" sensor similar to humans, its cognitive processes, emotional experiences, and decision-making qualities could take a significant step towards true artificial intelligence.

Methodology

Conceptual Framework

a. Gravity in Mind: Analogous to gravitational forces, this concept represents the cognitive pull or attraction towards certain decisions or judgments, akin to common sense in human cognition.

b. Game Refinement Theory: This theory provides a mathematical model to quantify the dynamics of engagement and fairness in games. It introduces parameters such as velocity (success rate), acceleration (GR value), and jerk (AD value) to describe the progression of games and the cognitive responses they elicit.

Empirical Data Collection

The empirical data for this study were collected from a variety of game types, including board games, such as Chess, Shogi, and Go. Additionally, key metrics were collected during each game session, including v as success rate, a as game acceleration and j as jerk.

Quantitative Analysis

The collected data were subjected to quantitative analysis to validate the conceptual framework and derive insights into the dynamics of mind engagement. The analysis involved statistical analysis, model fitting and comparison of games.

Model Validation

To ensure the validity and reliability of the proposed model, the cross-validation and comparison established models with steps were conducted.

By combining empirical data collection, quantitative analysis, and rigorous model validation, this study aims to advance our understanding of AI's potential to emulate human-like decision-making and common sense.

Discussion

Theoretical Implications

The "gravity in mind" mechanism offers a novel theoretical perspective on mind processes, drawing parallels between physical motion and mind engagement. Firstly, the gravity in mind mechanism integrates concepts from physics, game theory, and cognitive psychology, providing a unified framework to study dynamics of the mind. Secondly, by likening cognitive processes to free-fall motion, the gravity in mind mechanism offers insights into how common sense and decision-making operate, and the concepts of acceleration and jerk provide a quantitative basis to understand the fluctuations and stability in cognitive judgments, offering a new way to study the nuances of human cognition. Last but not least, this study demonstrates the value of an interdisciplinary approach, merging ideas from different fields to enhance our understanding of AI and the human mind. The integration of game refinement theory with cognitive processes opens new avenues for exploring how AI can mimic human-like reasoning and judgment.

Practical Applications

The practical applications of gravity in mind theory are extensive, particularly in the field of artificial intelligence. By incorporating the mechanism, AI systems can be designed to make more nuanced and human-like decisions. This could significantly improve AI's ability to handle complex and dynamic environments, such as strategic games, autonomous driving, and real-time decision-making systems. In addition, the gravity in mind theory could help bridge the gap between AI and human intelligence by providing AI with the ability of common sense. This would enable AI to better understand and predict human behaviour, enhancing its effectiveness in applications such as virtual assistants, customer service bots, and interactive learning platforms.

Conclusion

The "gravity in mind" theory offers a fresh perspective on understanding consciousness as an inherent essence, akin to gravitational forces shaping our reality. By uncovering the mechanisms behind this theory and exploring how they could be integrated into AI systems, we confront a profound question: Can AI move beyond mere functional capabilities to embody human-like discernment, empathy, and true intelligence?

While challenges undoubtedly exist, unconventional approaches like this have the potential to shed new light on the long-standing enigma of consciousness. They stimulate further interdisciplinary inquiry, bridging concepts from physics, philosophy, game theory, and AI research. As we continue grappling with this profound mystery, the harmonization of diverse viewpoints may prove instrumental in closing the gap between human and artificial cognition.

Ultimately, the quest for artificial general intelligence demands a synthesis of insights from various fields. It requires a willingness to challenge long-held assumptions and a recognition that the path to true intelligence may not lie in simply replicating human cognition. Instead, it may involve unveiling the fundamental essence that shapes our experiences and judgments — the elusive "gravity in mind."

Based upon the findings of the study, this study has identified several research directions, such as refinement of the gravity in mind model, which involves exploring more complex cognitive tasks and testing the model in diverse real-world scenarios. Investigating the practical integration of the gravity in mind mechanism into AI systems is a crucial next step. This includes developing algorithms and architectures that can effectively use the mechanism to enhance AI's decision-making and common-sense capabilities. Future research could also explore the impact of AI systems equipped with the gravity in mind mechanism on human-AI interaction. This involves studying how such systems influence user satisfaction, trust, and engagement in various contexts.

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Conflict of Interest Statement

The authors declare that they have no conflicts of interest to report regarding the present study.

References

[1] Turing, A. M. (1950). *Computing Machinery and Intelligence (1950)*. Mind, 59, 433-460. https://doi.org/10.1093/mind/LIX.236.433

[2] Iida, H., & Khalid, M. N. A. Using games to study law of motions in mind. In *IEEE Access, Vol. 8*, pp. 138701-138709. https://ieeexplore.ieee.org/stamp/stamp.jsp?tp=&arnumber=9151140

[3] Iida, H., Takahara, K., Nagashima, J., Kajihara, Y. & Hashimoto, T. (2004). An application of game-refinement theory to mah jong. In Rauterberg, M. (Ed.) *Entertainment Computing – ICEC 2004. ICEC 2004. Lecture Notes in Computer Science, Vol. 3166.* Springer Berlin Heidelberg. https://doi.org/10.1007/978-3-540-28643-1_41

[4] Khalid, M. N. A. & Iida, H. Objectivity and subjectivity in games: Understanding engagement and addiction mechanism. *IEEE Access*, Vol. 9, pp. 65187-65205. https://doi.org/10.1109/ACCESS.2021.3075954 (2021).

[5] Gao, N., Chang, H., Zhang, Z., Khalid, M. N. A. & Iida, H. (2022). Implications of jerk's on the measure of game's entertainment: Discovering potentially addictive games. In *IEEE Access*, *Vol. 10*, pp. 134362-134376. https://doi.org/10.1109/ACCESS.2022.3232520.

[6] Galilei, G., Crew, H. & de Salvio, A. (1914). *Dialogues concerning two new sciences*. University Studies Series. Macmillan Company.

[7] Guicciardini, N. (1687). Chapter 5 - Isaac Newton, Philosophiae naturalis principia mathematica, first edition (1687). In Grattan-Guinness, I., Cooke, R., Corry, L., Crépel, P. & Guicciardini, N. (Eds.), *Landmark writings in western mathematics 1640-1940* (pp. 59-87). https://doi.org/10.1016/B978-044450871-3/50086-3

[8] Sauer, T. Chapter 63 - Albert Einstein, review paper on general relativity theory (1916). In Grattan-Guinness, I., Cooke, R., Corry, L., Crépel, P. & Guicciardini, N. (Eds.), Landmark writings in western mathematics 1640-1940 (pp. 802-822). https://doi.org/10.1016/B978-044450871-3/50144-3

[9] Schick, F. Freedom and necessity: the transition to the logic of the concept in Hegel's science of logic. *Hegel Bulletin*, *35*(1), 84-99. https://doi.org/10.1016/B978-044450871-3/50144-3

[10] Zhang, Z., Xiaohan, K., & Khalid, M. N. A. & Iida, H. (2021). Bridging ride and play comfort. *Information*, *12*, 119. https://doi.org/10.3390/info12030119.

[11] Minsky, M. (2000). Commonsense-based interfaces. *Communications of the ACM*, *43*(8), 66-73. https://doi.org/10.1145/345124.345145