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BIFURCATION ANALYSIS FOR AROWANA FISH MODEL WITH HARVESTING EFFECT

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ABSTRACT

Wild Asian Arowana fish have been an endangered species since 1976, and need attention to avoid extinction. Factors that threaten the population of wild Arowana include its own reproductive method and spawning location. This research is aimed at considering a mathematical model to understand r the population dynamics of wild Arowana fish and its prey. The model is analysed both analytically and numerically. We solved the model to obtain equilibria and analysed the stability of equilibria by determining the eigenvalues of the Jacobian Matrix. The bifurcation analysis was also performed, in which the harvesting rate has been chosen as a critical parameter. The results found three equilibrium points, and the stability condition of these equilibria was analysed. It turned out that the model undergoes a transcritical bifurcation. Time series and phase portrait were also plotted to see the changes of dynamics for both populations for different values of harvesting parameter. Thus, this research is important to raise awareness on the need to control fishing behaviour so that the Arowana population can be sustained in the future.

2020 Mathematics Subject Classification: 37G05, 37C75

INTRODUCTION

The Asian Arowana fish (*Scleropages formosus*), also called "dragon fish", is one of the most valuable aquarium fish in the world. The name "dragon fish" comes from its long, narrow body with enormous scales and long, whisker-like chin barbels. In Chinese culture, it is considered a most auspicious fish that brings good luck and prosperity to its owner. The Chinese also believe Arowana are descended from a mythical dragon, which are highly valued in the culture [1]. The average lifespan of pet Arowana is about 10 to 15 years, and co go up to 20 years if the owner take exceptionally good care of it. The lifespan of wild Arowana are believed to be more than 20 years [2]. Many researchers have studied Arowana [3-5].

According to Yue *et al*. [6], this species is highly endangered and has been listed under Appendix I of CITES since 1975. Even though there are legal fish farms that breed Asian Arowana, the population trend of the species is still decreasing. Other than the overfishing and habitat loss in the mid-70s, there are many other factors that threaten Asian Arowana, such as its reproductive method and its spawning in open water.

Numerous studies have examined mathematical models for fish harvesting. Zhang *et al*. [7] examined a model of prey dispersal in a two-patch ecosystem, where there is supposed to be a zone designated for free fishing and another for reserved use, where fishing and other extractive activities are prohibited. They only address the dynamics of the system in the closed first quadrant in their article. They solved the algebraic equations that come from equating the left-hand derivatives to zero in order to discover the equilibria. As a result, they were able to determine three potential equilibria, two of which involved no predators.

Meanwhile, Lv *et al*. [8] considered a prey-predator model with harvesting for fishery resources within a reserve area. They solved the model to find the equilibria and analysed the stability of then equilibria, then analysed the bionomic equilibrium. They also discussed the optimal harvesting policy. Three equilibria were found and stability analysis was performed on the equilibria using Jacobian Matrix. The authors gave stability criteria of the model both from the local and global points of view. They found that over-harvesting would result in extinction of the population in the absence of the reserved zone, but both prey and predator species will coexist within a reserved zone because of the sustainability of the system.

Triharyuni and Aisyah [9] formulated a prey-predator model with harvesting of predator species, namely the interaction between Arowana fish with small fish. They found that the maximum harvest rate of Arowana was 0.0615 and the simulation indicated that it was necessary to restrict the exploitation of predatory fish to maintain its sustainability. It has been suggested that a prey-predator model involving the harvesting of both species within a conservation area, be conducted.

In 2018, the Pontryagin's maximum concept was used in the fish prey-predator model with harvesting strategy by Belkhodja *et al*. [10] and Manna *et al*. [11]. It is interesting to note that the Manna *et al*. [11] model showed the schooling behaviour of fish populations that include both predator and prey. As a result, interactions between predators and prey happen at the boundaries of all populations. After computing equilibrium points, Hopf bifurcation in the system was discovered.

In 2019, the dynamics of a fisheries model with two prey and one predator were examined by Raymond *et al*. [12]. Nile perch was the predator, whereas cichlid and tilapia fish were the prey. All three populations subject to harvesting impact. Proof of both local and global stability was provided to demonstrate the system's resilience. Their research revealed that the three populations were sustainable only if overharvesting of tilapia and cichlid fish is avoided, as those populations are the primary drivers of the Nile perch fish population's expansion.

Laham *et al*. [13] discussed the fish harvesting management strategies using logistic growth model for tilapia fish. The two logistic models are constant harvesting and periodic harvesting. Periodic harvesting is the optimal harvesting approach for the chosen fish farm. In order to fulfil the demand for tilapia fish, these discoveries can help fish farms boost t supply.

In this study, we aimed to consider a simple model of prey-predator for wild Asian Arowana fish and its prey fish. This work is motivated by Triharyuni and Aisyah [9] in which we modified their model from nonlinear functional response to linear response [14]. Although the model presented here is a simplification of the previous model done by [9], the novelty of this research is highlighted by performing bifurcation analysis, which can supplement the previous work. The objectives of this research are to obtain equilibrium points and to analyse the stability of the equilibrium points. Additionally, we will perform bifurcation analysis for the modified model in which the harvesting parameters of the Arowana are varied.

Mathematical Model

We consider a mathematical model which is modified from Triharyuni and Aisyah [9]. By removing the ratio part, we obtain the following system of ordinary differential equation:

$$\begin{aligned}\frac{dx}{dt} &= rx\left(1-\frac{x}{k}\right) - \alpha xy,\\ \frac{dy}{dt} &= c\alpha xy - my - hy,\end{aligned} \tag{1}$$

where the symbols of the above equations are described in Table 1. Concurrently, the flow diagram in Figure 1 shows the illustration for model (1). All parameters considered are positive constants.

Table 1: Description of variables and parameters of model (1)

Variable	Description
x	The prey fish population
y	The Arowana fish population
Parameter	**Description**
r	Growth rate of prey fish
k	Carrying capacity of both species
m	Mortality rate coefficient of Arowana
c	Conversion factor from the number of calories required by the new Arowana fish for each prey small fish caught
α	Coefficient prey fish predation by Arowana fish
h	Harvesting rate of Arowana fish

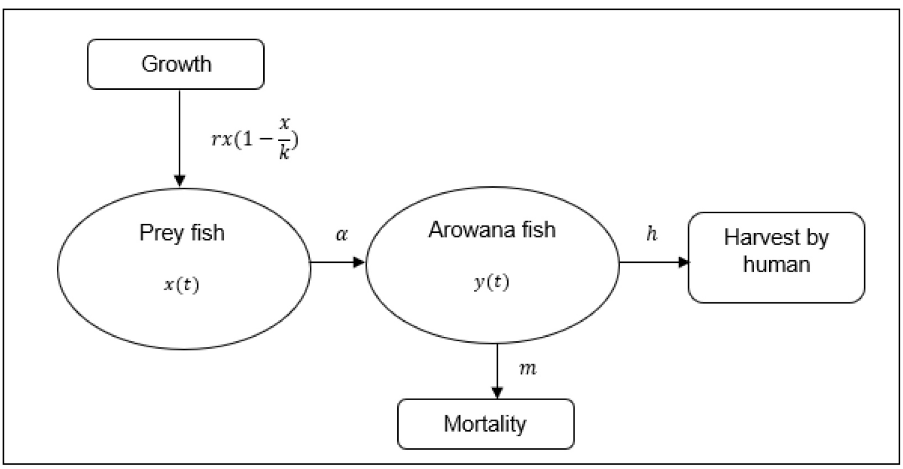

Figure 1: Interaction diagram between Arowana and prey fish

Equilibria Analysis

Equilibrium points are useful for predictions in mathematical modelling. A dynamical system might have more than one equilibrium point. Generally, a nonlinear system produces more than one equilibrium point. The more equilibrium points (i.e., equilibria) we have, the more complex behaviour dynamics we could obtain. Our research considers a nonlinear system of prey-predator interaction model.

To obtain the equilibria, we solve the system by letting the system (1) equal to zero.

$$\frac{dx}{dt} = rx\left(1 - \frac{x}{k}\right) - \alpha xy = 0$$

$$\frac{dy}{dt} = c\alpha xy - my - hy = 0$$

Therefore, we found a total of three equilibrium points:

$$E_1 = (0,0),$$

$$E_2 = (k, 0),$$

$$E_3 = \left(\frac{m+h}{c\alpha}, \frac{r(c\alpha k - h - m)}{c\alpha k}\right).$$

The equilibrium point means extinction for both populations, while means only the population of small fish survive. E_1 and E_2 are called trivial equilibria. The only nontrivial equilibrium point is E_3, in which both populations live together in harmony.

Local Stability Analysis for Equilibria

In this section, we analyse the stability of the equilibria by using Jacobian Matrix and determining the eigenvalues. This step is important to reduce the nonlinear system to linear so that the model can be easily analysed.

The general Jacobian Matrix's formula is given by:

$$(x^*, y^*) = \begin{bmatrix} \frac{\partial \dot{x}}{\partial x} & \frac{\partial \dot{x}}{\partial y} & \frac{\partial \dot{y}}{\partial x} & \frac{\partial \dot{y}}{\partial y} \end{bmatrix}.$$

Hence, the Jacobian Matrix for model (1) is obtained as:

$$J(x^*, y^*) = \left[r - \frac{2rx^*}{k} - \alpha y^* - \alpha x^* \; c\alpha y^* \; c\alpha x^* - m - h\right].$$

Then, the Jacobian Matrix is evaluated at each equilibrium point. The Jacobian matrix for the first equilibrium point E_1 is:

$$J_{E_1} = \left[r - \frac{2r(0)}{k} - \alpha(0) - \alpha(0) \; c\alpha(0) \; c\alpha(0) - m - h\right],$$

$$J_{E_1} = [r \; 0 \; 0 \; -m - h].$$

To find the eigenvalue, we evaluate the determinant of $(J - A\lambda)$=0:

$$det\, det\left(J_{E_1} - \lambda I\right) = 0,$$

$$(r-\lambda)(-m-h-\lambda) = 0.$$

Therefore, $\lambda_1 = r$ and $\lambda_2 = -(m+h)$. Since, $\lambda_1 > 0$, therefore E_1 (0,0) is always unstable. This means that the extinction for both prey fish and Arowana fish will not happen in the future.

For the second equilibrium point, $E_2 = (k,0)$, the Jacobian matrix is:

$$J_{E_2} = \left[r - \frac{2r(k)}{k} - \alpha(0) \;\; -\alpha(k) \;\; c\alpha(0) \;\; c\alpha(k) - m - h \right],$$

$$J_{E_2} = [-r \;\; -\alpha k \;\; 0 \;\; c\alpha k - m - h\,],$$

By taking the determinant,

$$det\, det\left(J_{E_2} - \lambda I\right) = 0,$$

$$(-r-\lambda)(c\alpha k - m - h - \lambda) = 0.$$

Therefore, $\lambda_1 = -r$ and $\lambda_2 = c\alpha k - (m+h)$. For this equilibrium point to be stable, both λ_1 and λ_2 must be negative, which implies $\lambda_1 < 0$ and $\lambda_2 < 0$. So, $c\alpha k - (m+h) < 0$ and the condition $c\alpha k < (m+h)$ must be fulfilled. Otherwise, if $c\alpha k > (m+h)$, then E_2 will be unstable. E_2 $(k, 0)$ means the small fish will grow until carrying capacity while Arowana fish become extinct. The stability of E_2 is given by:

i. if $r > 0$ and $c\alpha k < (m+h)$, then E_2 is asymptotically stable,

ii. if $r < 0$ and $c\alpha k > (m+h)$, then E_2 is unstable,

iii. and if r > 0 and $c\alpha k > (m+h)$, or $r < 0$ and $c\alpha k < (m+h)$, then E_2 is saddle where saddle implies unstable.

Finally, the Jacobian matrix for can be given as follows:

$$J_{E_3} = \left[r - \frac{2r\left(\frac{m+h}{c\alpha}\right)}{k} - \alpha\left(\frac{r}{\alpha}\left(1 - \frac{m+h}{c\alpha k}\right)\right) \;\; -\alpha\left(\frac{m+h}{c\alpha}\right) \;\; c\alpha\left(\frac{r}{\alpha}\left(1 - \frac{m+h}{c\alpha k}\right)\right) \;\; c\alpha\left(\frac{m+h}{c\alpha}\right) - m - h \right].$$

By taking the determinant:

$$det\, det\left(J_{E_3} - \lambda I\right) = 0,$$

$$det\, det\left(\left[-\frac{r(m+h)}{c\alpha k} \;\; -\frac{m+h}{c} \;\; rc\left(1 - \frac{m+h}{c\alpha k}\right) \;\; 0 \right] - \lambda[1\;0\;0\;1\,]\right) = 0,$$

$$det\, det\left(\left[-\frac{r(m+h)}{c\alpha k} - \lambda \;\; -\frac{m+h}{c} \;\; rc\left(1 - \frac{m+h}{c\alpha k}\right) \;\; -\lambda \right]\right) = 0,$$

$$\left(-\frac{r(m+h)}{c\alpha k} - \lambda\right)(-\lambda) - \left(rc\left(1 - \frac{m+h}{c\alpha k}\right)\left(-\frac{m+h}{c}\right)\right) = 0.$$

We obtain

$$\lambda_1 = \frac{1}{2}\left(\frac{r(-m-h)+\sqrt{A}}{c\alpha k}\right),$$

and

$$\lambda_2 = -\frac{1}{2}\left(\frac{r(m+h)+\sqrt{A}}{c\alpha k}\right),$$

where $A = -4c^2\alpha^2k^2hr - 4c^2\alpha^2k^2mr + 4c\alpha kh^2r + 8c\alpha km^2r + h^2r^2 + 2hmr^2 + m^2r^2$. We need to check for both λ_1 and λ_2 and as they can be both negative or positive values, or one of them is positive. For example, the condition to have a stable equilibrium point is that both λ_1 and λ_2 and must be negative. If one of them is positive, or both are positive, then E_3 will be unstable. For this third equilibrium, E_3 means that both small fish and Arowana fish will coexist or not depending on these conditions. The conditions of E_3 can be given as:

i. if $\lambda_1, \lambda_2 < 0$, then E_3 is asymptotically stable,
ii. if $\lambda_1, \lambda_2 < 0$, then E_3 is unstable,
iii. if $\lambda_1 > 0$ and $\lambda_2 < 0$, or $\lambda_1 < 0$ and $\lambda_2 > 0$, then E_3 is saddle, which implies unstable.

Global Stability Analysis for Coexistence Equilibrium Point E_3 (x*, y*)

In this section, we use the Lyapunov function approach to understand the global stability behaviour of the coexisting equilibrium E_3. One possible choice for the system (1) is a quadratic Lyapunov function:

$$V(x,y) = \frac{1}{2}(x^2 + y^2).$$

The derivative $V(x, y)$ of along the trajectories of the system can be computed as:

$$\frac{dV}{dt} = \frac{\partial V}{\partial x}\frac{dx}{dt} + \frac{\partial V}{\partial y}\frac{dy}{dt}$$

Thus,

$$\frac{dV}{dt} = (x)\left[rx\left(\frac{1-x}{K}\right) - \alpha xy\right] + (y)[c\alpha xy - my - hy].$$

Simplify the above to get:

$$\frac{dV}{dt} = x^2\left[r\left(\frac{1-x}{K}\right) - \alpha y\right] + y^2[c\alpha x - m - h],$$

i.e.

$$\frac{dV}{dt} = -x^2\left[\alpha y - r\left(\frac{1-x}{K}\right)\right] - y^2[h + m - c\alpha x].$$

Hence, $V(x, y)$ is a Lyapunov function provided that $\left[\alpha y - r\left(\frac{1-x}{K}\right)\right]$ and $[h + m - c\alpha x]$ are positive on some neighbourhood of coexisting equilibrium $E_3(x^*, y^*)$. With these conditions, we can conclude that the equilibrium E_3 is globally asymptotically stable.

NUMERICAL RESULTS

In this section, we discuss the numerical results of stability of equilibria for model (1). The software used are Maple and XPPAUT.

Stability of Equilibrium Point of Modified Model

Recall that for model (1), we obtained three equilibrium points, which are E_1 (0,0), $E_2 = (k,0)$, and $E_3 = \left(\frac{m+h}{c\alpha}, \frac{r}{\alpha}\left(1 - \frac{m+h}{c\alpha k}\right)\right)$. The set of parameter values used here were referred from [9] which are given by: $r = 0.8$, $k = 100$, $c = 0.75$, $m = 0.001$, $\alpha = 0.01$ and $h = 0.02$. Thus, the value of each equilibrium point after input of parameter values is given in Table 2.

Table 2: Equilibrium points of modified model

Equilibrium Point	The Prey Fish Population, x	The Arowana Fish Population, y
E_1	0	0
E_2	100	0
E_3	2.80	77.76

Since there is no negative value in any of these equilibrium points, hence we considered these three equilibrium points in the analysis.

The general Jacobian Matrix of the modified model is:

$$J(x^*, y^*) = \left[0.8 - \frac{16x^*}{1000} - 0.01y \;\; -0.01x^* \;\; 0.0075y^* \;\; 0.0075x^* - 0.021\right].$$

The eigenvalues obtained for each equilibrium point is shown in Table 3.

Table 3: Eigenvalues of equilibrium points

Equilibrium Point	Eigenvalue	Stability
E_1	$\lambda_1 = 0.800$	Unstable
E_2	$\lambda_2 = -0.021$ $\lambda_1 = 0.729$	Unstable
E_3	$\lambda_2 = -0.800$ $\lambda_1 = -0.0112 + 0.1272955616\,i$ $\lambda_2 = -0.0112 - 0.1272955616\,i$	Asymptotically stable

Since the eigenvalues for E_1 and E_2 have both positive and negative values, these two equilibrium points are unstable. As for E_3, since the eigenvalues are a pair of complex conjugate with negative real number parts (– 0.0112), it implies that the point is asymptotically stable. In the next section, we investigate the stability changes as the parameter of harvesting is varied.

Bifurcation Analysis for Harvesting Parameter

In this section, we employ bifurcation analysis to study the changes of stability for equilibria E_2 and E_3. Figure 2 shows the result of bifurcation diagram by using XPPAUT software for Arowana fish against harvesting parameter *h*. The red and black lines correspond to stable and unstable equilibria respectively. The range considered is for h ϵ [0,2]. This range is chosen since it is assumed that the highest rate of harvesting is 2. *h* = 2 means that every year, the number of Arowana fish captured is twice as much from previous year, due to high demand from humans.

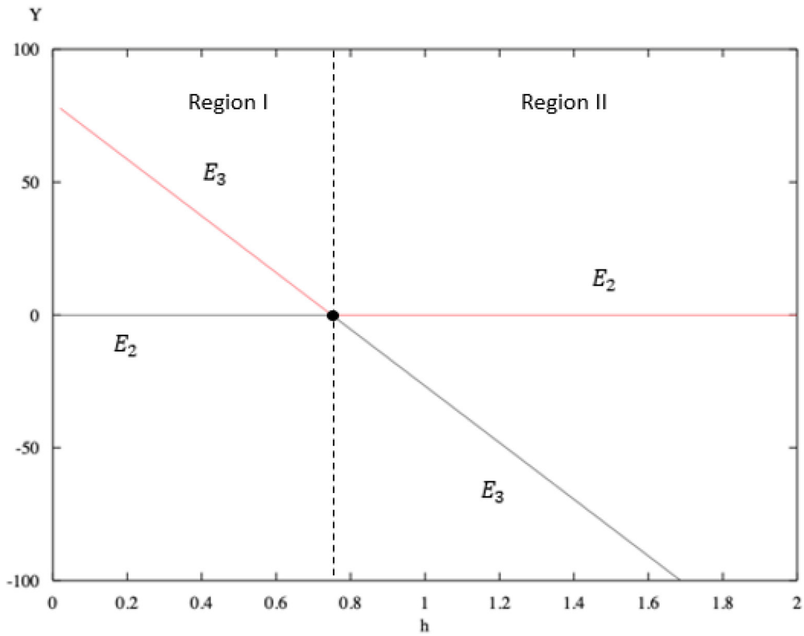

Figure 2: Bifurcation diagram of Arowana fish population (*y*) against harvesting rate (*h*)

In Figure 2, the bifurcation point occurred at $h = 0.749$ (marked by a full shaded circle). The diagram is divided into two Regions, I and II. In Region I ($0 < h \leq 0.749$), the coexistence equilibrium E_3 is stable. However, as the harvesting parameter increases, the Arowana population decreases. On the other hand, the equilibrium E_2 is unstable in this region.

Moving on to Region II, after the bifurcation point, for $0.749 < h \leq 2$, coexistence equilibrium is no longer stable. During this time, the extinction equilibrium is now stable. This means that if there is high harvesting effort by humans, the population of Arowana is zero, which indicates the extinction of this population. Therefore, since both equilibria change their stability at the bifurcation point, we call this type of bifurcation transcritical bifurcation. To clearly see the changes of stability, we show the results for different values of harvesting parameter in Table 4.

Table 4: Stability of equilibria for different values of harvesting parameter

Critical Parameter, *h*	**Equilibrium Point**	**Eigenvalues**	**Stability**
$h = 0.6$	$E_2 = (100,0)$	$\lambda_1 = 0.149$	Saddle
		$\lambda_2 = -0.8$	
	$E_3 = (80.13,15.89)$	$\lambda_1 = -0.23555$	Stable
		$\lambda_2 = -40552$	
$h = 0.7$	$E_2 = (100,0)$	$\lambda_1 = 0.049$	Saddle
		$\lambda_2 = -0.8$	
	$E_3 = (93.47,5.23)$	$\lambda_1 = -0.05272$	Stable
		$\lambda_2 = -69502$	
$h = 0.8$	$E_2 = (100,0)$	$\lambda_1 = -0.051$	Stable
		$\lambda_2 = -0.8$	
	$E_3 = (106.8, -5.44$	$\lambda_1 = 0.0482$	Saddle
		$\lambda_2 = -0.90267$	
$h = 0.9$	$E_2 = (100,0)$	$\lambda_1 = -0.151$	Stable
		$\lambda_2 = -0.8$	
	$E_3 = (120.13, -16.12)$	$\lambda_1 = 0.13268$	Saddle
		$\lambda_2 = -1.09375$	

After we input the critical parameter with different values, we observed that there are significant changes of stability for the equilibria. The critical parameter does not affect stability of first, but it shows that both second and third equilibria change their stability. We can conclude that as the harvesting rate of Arowana fish increases, the population of prey fish stablises while population of Arowana fish decreases. In other words, when harvesting rate is lower, the population of Arowana fish is stable and higher, and, therefore, affects the population size of prey fish due to the number of its predators increasing. We also show the dynamics for both populations in forms of time series and phase portrait plots in Figures 3 and 4, for low harvesting ($h = 0.6$) as well as for high harvesting ($h = 0.8$).

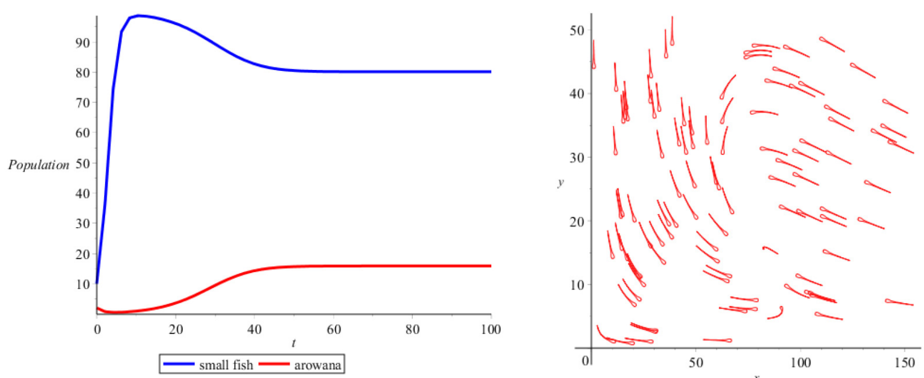

Figure 3: Time series and phase portrait for $h = 0.6$

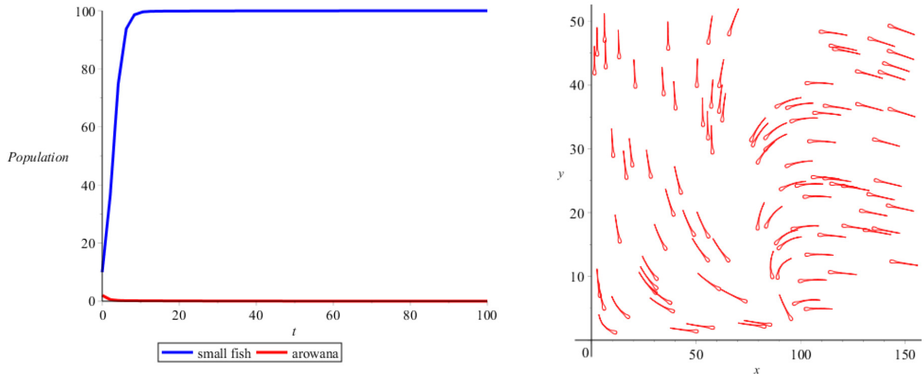

Figure 4: Time series and phase portrait for $h = 0.8$

Conclusion

We had modified the prey-predator model of Thiharyuni and Aisyah [9], and we obtained three equilibrium points. The stability for each equilibrium points had been analysed and it was proved that the survival or extinction of the Arowana population depends on certain conditions. Moreover, as we vary the harvesting rate on the Arowana, the results showed that the model undergoes a transcritical bifurcation, in which the stability changes between the survival and extinction equilibria. For low harvesting, the population of Arowana could survive, while for higher harvesting effort, the species will die out in the future. We also discussed the dynamics of our model using time series and phase portrait. They showed that the behaviour of prey fish and Arowana fish can influence the population size of each other.

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Conflict of Interest Statement

The authors declare that they have no conflict of interest.

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