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MAPPINGS AND DECOMPOSITIONS OF (μ, σ)-CONTINUITY ON μ-NEARLY COMPACT SPACES

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ABSTRACT

In generalised topological space (X,μ), a μ-space is said to be μ-nearly compact if every μ-regular open cover of X has a finite subcover. In literature, many generalised continuities in a topological space are constructed using many generalised open sets. Relationships among them are studied by proving their implications and finding counterexamples for their independent relationships. Many researchers called this type of study a decomposition of continuities. In this paper, we further investigate some decompositions of (μ,σ)-continuity and applying them to μ-nearly compact spaces. Moreover, we also analyse the effect of mappings on these spaces and obtain some results. The main result is that the (μ,σ)-δ-continuous image of μ-nearly compact space is σ-nearly compact.

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INTRODUCTION

In 2002, Császár [2] introduced and studied the concepts of generalised topological spaces (GTS) and continuous function, also related to the interior and closure operator on GTS. Since then, in recent years, many researchers have introduced several new concepts related to continuity in generalised topological spaces. Therefore, in 2008, Császár introduced and studied the concepts of δ-open set and some of its properties [3]. The concept of almost continuity in generalised topological spaces has been introduced by Min [8]. A generalised concept of continuous function defined by Császár is presented in this paper. Later, in 2010, Min [9] introduced (δ, δ')-continuity on generalised topological spaces, which is stronger and independent than almost (μ, λ)-continuity. The characterisations of (δ, δ')-continuous functions are also investigated. Moreover, the concept of (δ, δ')-continuity relative to several types of continuity is also studied and several results are obtained. Next, in 2010 Kılıçman and Salleh [13] also studied and gave further new results about mappings and decompositions of pairwise continuity on pairwise nearly Lindelöf spaces in a bitopological setting. Hence, most of their results have been extended from topological spaces to bitopological spaces.

Recently, in 2021, Salleh and Rashdi [14] studied the notion of μ-nearly compactness in generalised topological spaces. They also studied the fundamental properties of μ-nearly compact space. Furthermore, they continued to study the μ-nearly compact spaces in generalised topological space and the conditions for characterising the μ-nearly compact spaces. Lastly, they investigated the properties of μ-nearly compact subspaces and subsets of a GTS. Some counterexamples are demonstrated to establish that μ-nearly compact generalised topological space is not satisfying the hereditary property.

The purpose of the present paper is to study the decompositions of continuity concepts and the effect of mappings on μ-nearly compact spaces in generalised topological spaces (GTSs). Hence, in the next section, all preliminaries and basic definitions useful for this paper are collected. Then, the decompositions of (μ,σ)-continuity and some of its properties have been investigated. Later, we also analysed the effect of mapping on μ-nearly compact spaces. The main result of this paper is to show that the$(\mu,\sigma)\delta$-continuous image of μ-nearly compact space is σ-nearly compact.

PRELIMINARIES

In this paper, a space (X, μ) is always means generalised topological spaces (GTS). For a nonempty set X, let (X) denotes the power set of X. A collection μ of subsets of $exp(X)$ is called a generalised topology (briey GT) and (X, μ) is called a generalised topological space (GTS) [2] if $\emptyset \in \mu)$ and the union of elements of μ belong to μ. The elements of μ on GT are called μ-open sets and the complements are called μ-closed sets. If K is a subset of the space (X, μ), then, the μ-closure of K, denoted by $c\mu(K)$ is the intersection of all μ-closed sets containing K, that is, the smallest μ-closed set containing K; and the μ-interior of K, denoted by $i_\mu(K)$ is the union of all μ-open sets contained in K, that is, the largest μ-open set contained in K [2, 3]. The set is μ-open if and only if $K = i_\mu(K)$, and $c_\mu(K) = X \setminus \big(i_\mu(X \setminus K)\big)$. If the condition $X \in \mu$ is added, then the space (X, μ) is called μ-space [11]. By μ-open cover of X, we mean that the cover of X by μ-open subsets of X. Similarly, by the μ-regular open cover of X, we mean the cover of X by μ-regular open subsets of X.

In generalised topological spaces, the following concepts such as μ-open, μ-regular open, μ-regular closed, μ-preopen and μ-β-open sets are well known among the topologist. Therefore, these concepts are being studied by many researchers who are given in the following definition.

Definition 2.1. Let K be a subset of a GTS (X, μ). Then, K is called:

(a) μ-open [3] if $K = i_\mu(K)$;

(b) μ-regular open [15] if $K = i_\mu\big(c_\mu(K)\big)$;

(c) μ-regular closed [15] if $K = c_\mu\big(i_\mu(K)\big)$;

(d) μ-preopen [3] if $K \subseteq i_\mu\big(c_\mu(K)\big)$;

(e) μ-β open [3] if $K \subseteq c_\mu\big(i_\mu(c_\mu(K)\big)$.

Definition 2.2. [6] A μ-space (X, μ) is said to be μ-nearly compact if every μ-open cover $M = \{U_\alpha : \alpha \in I\}$ of X has a finite subcollection, $M' = \{U_{\alpha_1}, U_{\alpha_2},...,U_{\alpha_n}\}$ such that $X = \bigcup_{i=1}^{n} i_\mu c_\mu U_{\alpha_i}$.

DECOMPOSITION OF (μ,α)-CONTINUITY

The various concepts of continuous functions in a topological space such as R-map, almost continuous, precontinous, β-continuous, almost precontinuous, almost β-continuous, δ-continuous and almost δ-continuous functions have been introduced and gained attention by many researchers (see [8, 5]). These concepts are extended into generalised topological spaces as follows.

Definition 3.1. Let (X,μ) and (Y,σ) be GTS. A function $f:(X,\mu) \to (Y,\sigma)$ is said to be:

(a) (μ,σ)-continuous [2] if $f^{-1}(V)$ is μ-open set in X for every σ-open set V in Y ;

(b) (μ,σ)-R-map if $f^{-1}(V)$ is μ-regular open set in X for every σ-regular open set V in Y;

(c) (μ,σ)-almost continuous [8] if $f^{-1}(V)$ is μ-open set in X for every σ-regular open set V in Y;

(d) (μ,σ)-precontinuous (resp. (μ,σ)-β-continuous) if $f^{-1}(V)$ is μ-preopen (resp. μ-β-open) set in X for every σ-open set V in Y;

(e) (μ,σ)-almost precontinuous (resp. (μ,σ)-almost β-continuous) if for every $x \in X$ and every σ-regular open set V in Y containing $f(x)$, there exists a μ-preopen (resp. μ-β-open) set U in X containing such that $f(U)\subseteq V$;

(f) (μ,σ)-δ continuous (resp. (μ,σ)-almost δ-continuous) if for every $x \in X$ and every δ-regular open subset V of Y containing $f(x)$, there exists a μ-regular open subset U of X containing x such that $f(U) \subseteq V$ (resp. $f(U) \subseteq {}_{C_\sigma}(V)$.

It is very clear from the definition that for a function $f: (X,\mu) \to (Y,\sigma)$, if f is (μ,σ)-almost continuous, then, $f^{-1}(V)$ is μ-closed set in X for every σ-regular closed set V in Y .

Lemma 3.1. *Assume $\{U_\alpha : \alpha \in I\}$ is a collection of μ-β-open (resp. μ-preopen) sets in X. Then, $\bigcup_{\alpha\in I} U_\alpha$ is μ-β-open (resp. μ-preopen) set in X.*

Proof. We need to prove the μ-β-open part of the lemma. For μ-preopen, it can be proved similarly. Since for every $\alpha \in I$, U_α is μ-β-open set in X, we have $U_\alpha \subseteq c_\mu\left(i_\mu(c_\mu(U_\alpha))\right)$. Then,

$$\bigcup_{\alpha\in I} U_\alpha \subseteq \bigcup_{\alpha\in I} c_\mu\left(i_\mu\left(c_\mu(U_\alpha)\right)\right) \subseteq c_\mu\left(\bigcup_{\alpha\in I} i_\mu\left(c_\mu(U_\alpha)\right)\right)$$

$$\subseteq c_\mu\left(i_\mu\left(\bigcup_{\alpha\in I} c_\mu(U_\alpha)\right)\right) \subseteq c_\mu\left(i_\mu\left(c_\mu\left(\bigcup_{\alpha\in I} U_\alpha\right)\right)\right).$$

Hence, $\bigcup_{\alpha\in I} U_\alpha$ is μ-β-open set in X.

Theorem 3.1. *For a function $f: (X, \mu) \to (Y, \sigma)$, the following are equivalent:*

(1) *f is (μ, σ)-almost precontinuous (resp.(μ, σ)-almost β-continuous);*

(2) *$f^{-1}(V)$ is μ-preopen (resp. μ-β-open) set in X for every σ-regular open set V in Y.*

Proof. (1) ⇒ (2): Let V be any σ-regular open set in Y and let $x \in f^{-1}(V)$. Then, $f(x) \in V$, and by (1), there exists a μ-preopen set U_x in X containing x such that $f(U_x) \subseteq V$. Hence, $x \in U_x \subseteq f^{-1}(V)$. Therefore, we obtain $x \in i_u\left(c_u(f^{-1}(V))\right)$ by the definitions of μ-interior and μ-closure. This shows that $f^{-1}(V) \subseteq i_u\left(c_u(f^{-1}(V))\right)$ and thus, $f^{-1}(V)$ is μ-preopen set in X.

It also can be obtained from Lemma 3.1 since $x \in U_x \subseteq f^{-1}(V)$ and $f^{-1}(V) = U_{x\in f^{-1}(V)} U_x$.

(2) ⇒ (1): Let $x \in X$ and let V be σ-regular open set in Y containing $f(x)$. Then, $\in f^{-1}(V)$ and by (2), $f^{-1}(V)$ is μ-preopen set in X. So, take $\cup = f^{-1}(V)$, then, U is a μ-preopen set in X containing x such that $f(U) = f\left(f^{-1}(V)\right) \subseteq V$. This shows that f is (μ,σ)-almost precontinuous.

The (μ,σ)-almost β-continuous can also be proved in a similar way.■

Proposition 3.1. *If a function* $f:(X, \mu) \rightarrow (Y, \sigma)$ *is a* (μ, σ)*-almost continuous, then, f is* (μ, σ)*-almost* δ *-continuous.*

Proof. Let $x \in X$ and let V be a σ-regular open set in Y containing $f(x)$. Then, $\in f^{-1}(V)$ and since f is (μ, σ)-almost continuous, $f^{-1}(V)$ is a μ-open set in X containing x. Since $= i_u\left(c_u\left(f^{-1}(V)\right)\right)$ is a μ-regular open set in X containing x,

$$f(Z) = f\left(i_u(c_u\,(f^{-1}(V))\right) \subseteq f\left(c_u\,(f^{-1}(V))\right).$$

Since f is also (μ, σ)-almost continuous and $c_\sigma(V)$ is a σ-regular closed set in Y, then:

$$c_u\left(f^{-1}(V)\right) \subseteq f^{-1}\left(c_\sigma(V)\right)$$

because $f^{-1}\left(c_\sigma(V)\right)$ is a μ-closed set in X containing $f^{-1}(V)$. So:

$$f(Z) \subseteq f\left(c_u\,(f^{-1}(V))\right) \subseteq f\left(f^{-1}\left(c_\sigma(V)\right)\right) \subseteq c_\sigma(V).$$

Thus, f is (μ, σ)-almost δ-continuous.■

The converse of Proposition 3.1 is not true, as shown in the following counterexample.

Example 3.1. Let $X = \{x, y, z\}$ and assume the generalised topologies on X are $\mu = \{\emptyset\{y, z\}, X\}$ and $\sigma = \{\emptyset\{x\},\{y\},\{x, y\}, X\}$. Let $f: \{X, \mu\} \rightarrow \{X,\sigma\}$ be a function defined by $f(x) = y$ and $f(y) = f(z) = z$. Then, f is (μ, σ)-almost δ-continuous but it is not (μ, σ)-almost continuous since $f^{-1}(\{y\}) = \{x\}$ is not μ-open set in $\{X, \mu\}$ and $\{y\}$ is σ-regular open set in (X, σ). Hence, f is not (μ, σ)-almost continuous. Even f is (μ, σ)-almost δ-continuous, it is not (μ, σ)-δ-continuous since for the σ-regular open set $\{y\}$ in (X, σ) containing $f(x) = y$, there is no μ-regular open set U in (X, μ) containing x such that $f(U) \subseteq \{y\}$. It is also not (μ, σ)-continuous since $f^{-1}(\{y\}) = \{x\}$ is not μ-open in (X, μ) while $\{y\}$ is σ-open in (X, σ).

In the following Proposition 3.2, a (μ, σ)-δ-continuity implies (μ, σ)-almost continuity but the converse is not true as shown in Example 3.2 below.

Proposition 3.2. *If a function* $f:(X, \mu) \rightarrow (Y, \sigma)$ *is a* (μ, σ)-δ-*continuous, then, f is* (μ, σ)*-almost continuous.*

Proof. Let V be a σ-regular open set in Y and let $x \in f^{-1}(V)$. Then, $(x) \in V$, and since f is (μ, σ)-δ-continuous function, there is a μ-regular open set U_x in X containing x such that $f(U_x) \subseteq V$. Then, $x \in U_x \subseteq f^{-1}(V)$. As we know, every μ-regular open set is μ-open, we will have U_x is μ-open in X for each x. Hence, $x \in i_u\left(f^{-1}(V)\right)$ and thus, $f^{-1}(V) = i_u\left(f^{-1}(V)\right)$, i.e., $f^{-1}(V)$ is μ-open set in X. Thus, f is (μ, σ)-almost continuous.■

Note that every (μ, σ)-continuous function is (μ, σ)-almost continuous and every (μ, σ)-R-map is also (μ, σ)-almost continuous, but the converses are not true in general. In fact, the concepts of (μ, σ)-continuity and (μ, σ)-R-map are independent as shown in Examples 3.2 and 3.3 below. Furthermore, the concepts of (μ, σ)-continuity and (μ, σ)-δ-continuity, as shown in Examples 3.2 and 3.3 are independent too. By Lemma 4.1, every (μ, σ)-R-map is (μ, σ)-δ-continuous, but the converse is not generally true, as shown in Example 4.1 below. In addition, the concepts of (μ, σ)-continuity and (μ, σ)-almost δ-continuity are also independent, as shown in Example 3.1 above and Example 3.2 below. Clearly, every (μ, σ)-δ-continuity is (μ, σ)-almost δ-continuity but the converse is not true in general, as shown in Example 3.1 above. The concepts for (μ, σ)-almost continuity and (μ, σ)-almost δ-continuity are independent, as shown in Example 3.1 and Example 3.2 below.

Example 3.2. Let $X = \{x, y, z\}$ with generalised topologies $\mu = \{\emptyset\{x\},\{y\},\{x, y\},\{y, z\}, X\}$ and σ $\{\emptyset\{x\},\{z\},\{x, z\}\}$ on X. Let f: $(X, \mu)\rightarrow(X, \sigma)$ be a function defined by $f(x) = z$, $f(y) = x$ and $f(z) = y$. Then, f is (μ, σ)-continuous so (μ, σ)-almost continuous. But f is not (μ, σ)-R-map since $f^{-1}(\{x\}) = \{y\}$ is not μ-regular open set in (X, μ) while $\{x\}$ is a σ-regular open set in (X, σ). Even f is (μ, σ)-continuous and (μ, σ)-almost continuous, it is not (μ, σ)-almost δ-continuous since $\{x\}$ is σ- regular open set in (X, σ) containing $f(y) = x$ but there is no μ-regular open set U in (X, μ) containing y such that $f(U) \subseteq c_\sigma(\{x\}) = \{x,y\}$. Hence, f is also not (μ, σ)-δ-continuous.

Example 3.3. Let $X = \{x, y, z\}$ with generalised topologies $\mu = \{\emptyset\{z\},\{x, y\}, X\}$ and $\sigma = \{\emptyset\{x\},\{x, y\}\}$ on X. Let f: $(X, \mu) \rightarrow (X, \sigma)$ be the identity function. Then, f is a (μ, σ)-R-map since Ø and $\{x, y\}$ are the only σ-regular open set in (X, σ). So, f is (μ, σ)-δ-continuous and also (μ, σ)-almost continuous. But f is not (μ, σ)-continuous since $f^{-1}(\{x\}) = \{x\}$ is not μ-open set in (X, μ) and is σ-open set in (X, σ).

Therefore, we obtain the following diagram (Figure 1) in which none of these implications is reversible.

(μ, σ)-R-map

$\Downarrow$

(μ, σ)-δ-continuous

$\Downarrow$

(μ, σ)-continuous $\Rightarrow$ (μ, σ)-almost continuous

$\Downarrow$

(μ, σ)-almost δ-continuous

Figure 1: Diagram

Several types of open functions between topological spaces are studied by many authors such as almost open, almost α-open, weakly open and M-preopen (see [5, 10, 12]). We extend these types of open functions into generalised topological spaces as follows.

Definition 3.2. Let $(X, \mu) \rightarrow (Y, \sigma)$ and be GTS. A function f: $(X, \mu) \rightarrow (Y, \sigma)$ is said to be:

(a) (μ, σ)-open if $f(U)$ is σ-open set in Y for every μ-open set U in X;

(b) (μ, σ)-almost open [1] if $f(U)$ is σ-open set in Y for every μ-regular open set U in X;

(c) (μ, σ)-almost α-open if $f(U) \subseteq i_\sigma\left(c_\sigma(i_\sigma(f(U)))\right)$ for every μ-regular open set U in X;

(d) (μ, σ)-weakly open [7] if $f(U) \subseteq i_\sigma\left(f(c_\mu(U))\right)$ for every μ-open subset U of X;

(e) (μ, σ)-M-preopen if $f(U)$ is σ-preopen set in Y for every μ-preopen set U in X.

In the following proposition, we prove that (μ, σ)-almost open function implies (μ, σ)-weakly open, as shown below.

Proposition 3.3. *Assume that a function f:* $(X, \mu) \rightarrow (Y, \sigma)$ *is* (μ, σ)*-almost open, then, f is* (μ, σ)*-weakly open.*

Proof. Let U be a μ-open subset of X. Then, $i_\mu(c_\mu(U))$ is a μ-regular open subset of X. Since f is (μ, σ)-almost open, then, $f(i_\mu(c_\mu(U)))$ is a σ-open set in Y. Therefore, $f(i_\mu(c_\mu(U))) = i_\sigma(f(i_\mu(c_\mu(U))))$ $\subseteq i_\sigma(f(c_\mu(U)))$. Since $U \subseteq i_\mu(c_\mu(U))$, it implies that $f(U) \subseteq f(i_\mu(c_\mu(U)))$. Hence, $f(U) \subseteq i_\sigma(f(c_\mu(U)))$. This shows that f is (μ, σ)-weakly open.■

The converse of Proposition 3.3 is not true in general (see [15]).

MAPPING ON μ-NEARLY COMPACT SPACES

In this section, we studied some properties of (μ, σ)-continuous function and (μ, σ)-open function on μ-nearly compact spaces in generalised topological spaces. However, firstly we show that if f: (X, μ) $\rightarrow (Y, \sigma)$ is surjective and (μ, σ)-δ-continuous, then, the property of near compactness is preserved.

Definition 4.1 [6]. A μ-space (X, μ) is said to be μ-nearly compact if for every μ-open cover $M=\{U_\alpha: \alpha \in I\}$ of X contains a finite subcollection $M' = \{U_{\alpha_1}, ..., U_{\alpha_n}\}$ such that $X = \bigcup_{j=1}^{n} i_\mu(c_\mu(U_{\alpha j}))$ or equivalently, if every μ-regular open cover of has a finite subcover.

Theorem 4.1. *Assume that a function f:* $(X, \mu) \rightarrow (Y, \sigma)$ *is a surjective and* (μ, σ)*-*δ*-continuous function. If X is μ-nearly compact, then, Y is σ-nearly compact.*

Proof. Let $\{V_\alpha : \alpha \in I\}$ be a σ-regular open cover of Y. Let $x \in X$ and let $\alpha_x \in I$ such that $f(x) \in V_{\alpha_x}$. Since f is (μ, σ)-δ-continuous, there is a μ-regular open subset U_{α_x} of X containing x such that $f(U_{\alpha_x})$ $\subseteq V_{\alpha_x}$. Therefore, $\{U_{\alpha_x} : x \in X\}$, forms a μ-regular open cover of X. Since X is μ-nearly compact, there is a finite subcollection $\{U_{\alpha_{x_1}}, ..., U_{\alpha_{x_n}}\}$ such that $X = \bigcup_{j=1}^{n} U_{\alpha_{x_1}}$. Since f is surjective, then, we have

$$Y = f(X) = f\left(\bigcup_{i=1}^{n} U_{\alpha_{x_i}}\right) = \bigcup_{i=1}^{n} f\left(U_{\alpha_{x_i}}\right) \subseteq \bigcup_{i=1}^{n} V_{\alpha_{x_i}}$$

which implies $Y = \bigcup_{j=1}^{n} V_{\alpha_{x_i}}$. This shows that Y is σ-nearly compact. ■

Lemma 4.1. *If a function f:* $(X, \mu) \rightarrow (Y, \sigma)$ *is a* (μ, σ)*-R-map, then, f is* (μ, σ)*-*δ*-continuous.*

Proof. Let $x \in X$ and let V be a σ-regular open subset of Y containing $f(x)$. Then, $x \in f^{-1}(V)$. Since f is a (μ, σ)-R-map, $f^{-1}(V)$ is a μ-regular open set in X. So, if $= f^{-1}(V)$, then, U is a μ-regular open subset of X containing x such that $f(U) = f(f^{-1}(V)) \subseteq V$. This implies that f is (μ, σ)-δ-continuous. ■

The converse of Lemma 4.1 is not true, as shown in the following example.

Example 4.1. Let $X = \{a, b, c\}$ and $Y = \{x, y, z\}$. We assume the generalized topology $\mu = \{\emptyset, \{a\}, \{b\}, \{a,b\}, X\}$ on X and assume the generalised topology $\sigma = \{\emptyset, \{y\}, \{z\}, \{y,z\}, Y\}$ on Y. Let f: $(X, \mu) \rightarrow (Y, \sigma)$ be a function defined by $f(a) = f(b) = y$ and $f(c) = x$. Then, f is (μ, σ)-δ-continuous but it is not (μ, σ)-R-map since $f^{-1}(\{y\}) = \{a,b\}$ is not μ-regular open set in (X, μ) and $\{y\}$ is σ-regular open set in (Y, σ).

Using Lemma 4.1 and Theorem 4.1 as shown above, we have the following corollary.

Corollary 4.1. *Let f: $(X, \mu) \to (Y, \sigma)$ be a surjective and (μ, σ)-R-map. If X is μ-nearly compact, then, Y is σ-nearly compact.*

Lemma 4.2. *Every (μ, σ)-almost continuous and (μ, σ)-almost α-open function is a (μ, σ)-R-map.*

Proof. Let f: $(X, \mu) \to (Y, \sigma)$ be a (μ, σ)-almost continuous and (μ, σ)-almost α-open function. Let V be a σ-regular open set in Y. Since f is (μ, σ)-almost continuous, $f^{-1}(V)$ is a μ-open set in X. Thus, $f^{-1}(V) \subseteq i_u\left(c_u(f^{-1}(V))\right)$. Next, we will show the opposite inclusion. Since f is (μ, σ)-almost α-open and $i_u\left(c_u(f^{-1}(V))\right)$ is a μ-regular open set in X, then, we have:

$$f^{-1}\left(i_\mu\left(c_\mu(f^{-1}(V))\right)\right) \subseteq i_\sigma\left(c_\sigma\left(i_\sigma\left(f\left(i_\mu\left(c_\mu(f^{-1}(V))\right)\right)\right)\right)\right)$$
$$\subseteq i_\sigma\left(c_\sigma\left(i_\sigma\left(f\left(c_\mu(f^{-1}(V))\right)\right)\right)\right).$$

Since f is (μ, σ)-almost continuous and $c_\sigma(V)$ is a σ-regular closed set in Y, $c_u(f^{-1}(V)) \subseteq f^{-1} c_\sigma(V))$ because $f^{-1}\left(c_\sigma(V)\right)$ is a μ-closed set in X containing $f^{-1}(V)$. Thus,

$$f\left(i_\mu\left(c_\mu(f^{-1}(V))\right)\right) \subseteq i_\sigma\left(c_\sigma\left(i_\sigma\left(f\left(c_\mu(f^{-1}(V))\right)\right)\right)\right) \subseteq i_\sigma\left(c_\sigma\left(i_\sigma\left(f\left(f^{-1}(c_\sigma(V))\right)\right)\right)\right)$$
$$\subseteq i_\sigma\left(c_\sigma\left(i_\sigma(c_\sigma(V))\right)\right) \subseteq i_\sigma(c_\sigma(V)) = V.$$

Therefore,

$$i_\mu\left(c_\mu(f^{-1}(V))\right) \subseteq f^{-1}\left(f\left(i_\mu\left(c_\mu(f^{-1}(V))\right)\right)\right) \subseteq f^{-1}(V).$$

Hence, $f^{-1}(V) = i_u\left(c_u(f^{-1}(V))\right)$ which implies that $f^{-1}(V)$ is a μ-regular open set in X. This shows that f is a (μ, σ)-R-map. ■

Corollary 4.2. *Let f: $(X, \mu) \to (Y, \sigma)$ be a surjective, (μ, σ)-almost continuous and (μ, σ)-almost α-open function. If X is μ-nearly compact, then, Y is σ-nearly compact.*

Proof. This consequence follows from Lemma 4.2, Lemma 4.1 and Theorem 4.1 above.

Lemma 4.3. *Every (μ, σ)-almost continuous and (μ, σ)-weakly open function is a (μ, σ)-R-map.*

Proof. Let f: $(X, \mu) \to (Y, \sigma)$ be a (μ, σ)-almost continuous and (μ, σ)-weakly open function. Let V be a σ-regular open set in Y. Since f is (μ, σ)-almost continuous, $f^{-1}(V)$ is a μ-open set in X. Thus, $f^{-1}(V) \subseteq i_u\left(c_u(f^{-1}(V))\right)$. Next, we will show the opposite inclusion. Since, f is (μ, σ)-weakly open and $i_u\left(c_u(f^{-1}(V))\right)$ is also μ-open set in X, then, we have:

$$f\left(i_\mu\left(c_\mu(f^{-1}(V))\right)\right) \subseteq i_\sigma\left(f\left(c_\mu\left(i_\mu\left(c_\mu(f^{-1}(V))\right)\right)\right)\right) \subseteq i_\sigma\left(f\left(c_\mu(f^{-1}(V))\right)\right).$$

Since f is (μ, σ)-almost continuous, $c_u(f^{-1}(V)) \subseteq f^{-1}(c_\sigma(V))$. Hence,

$$f\left(i_\mu\left(c_\mu(f^{-1}(V))\right)\right) \subseteq i_\sigma\left(f\left(c_\mu(f^{-1}(V))\right)\right) \subseteq i_\sigma\left(f\left(f^{-1}(c_\sigma(V))\right)\right) \subseteq i_\sigma(c_\sigma(V)) = V.$$

Therefore,

$$i_\mu\left(c_\mu(f^{-1}(V))\right) \subseteq f^{-1}\left(f\left(i_\mu\left(c_\mu(f^{-1}(V))\right)\right)\right) \subseteq f^{-1}(V).$$

Hence, $f^{-1}(V) = i_u(c_u(f^{-1}(V)))$ which implies that $f^{-1}(V)$ is a μ-regular open set in X. This shows that f is a (μ, σ)-δ-map.■

By using Lemma 4.3, Lemma 4.1 and Theorem 4.1 as above, we conclude the following corollary.

Corollary 4.3. *Let f: $(X, \mu) \rightarrow (Y, \sigma)$ be a surjective, (μ, σ)-almost continuous and (μ, σ)-weakly open function. If X is μ-nearly compact, then, Y is σ-nearly compact.*

Lemma 4.4. *Every (μ, σ)-almost continuous and (μ, σ)-M-preopen function is (μ, σ)-R-map.*

Proof. Let f: $(X, \mu) \rightarrow (Y, \sigma)$ be a (μ, σ)-almost continuous and (μ, σ)-M-preopen function. Let V be a σ-regular open set in Y. Since f is (μ, σ)-almost continuous, $(f^{-1}(V)$ is a μ-open set in X. Thus, $f^{-1}(V) \subseteq i_u(c_u(f^{-1}(V)))$. Next, we will show the opposite of inclusion. Since $i_u(c_u(f^{-1}(V))) \subseteq i_u(c_u(i_u(c_u(f^{-1}(V)))))$, i.e., $i_u(c_u(f^{-1}(V)))$ is a μ-preopen set in X, and f is (μ, σ)-M-preopen, then, $f(i_u(c_u(f^{-1}(V)))$ is a σ-preopen set in Y, i.e.,

$$f\left(i_\mu\left(c_\mu(f^{-1}(V))\right)\right) \subseteq i_\sigma\left(c_\sigma\left(f\left(i_\mu\left(c_\mu(f^{-1}(V))\right)\right)\right)\right) \subseteq i_\sigma\left(c_\sigma\left(f\left(c_\mu(f^{-1}(V))\right)\right)\right).$$

Since f is (μ, σ)-almost continuous, $c_\mu(f^{-1}(V)) \subseteq f^{-1}(c_\sigma V))$. So,

$$f\left(i_\mu\left(c_\mu(f^{-1}(V))\right)\right) \subseteq i_\sigma\left(c_\sigma\left(f\left(f^{-1}(c_\sigma(V))\right)\right)\right) \subseteq i_\sigma\left(c_\sigma(c_\sigma(V))\right) \subseteq i_\sigma(c_\sigma(V)) = V.$$

Therefore,

$$i_\mu\left(c_\mu(f^{-1}(V))\right) \subseteq f^{-1}\left(f\left(i_\mu\left(c_\mu(f^{-1}(V))\right)\right)\right) \subseteq f^{-1}(V).$$

Hence, $f^{-1}(V) = i_\mu(c_\mu(f^{-1}(V)))$ which implies that $f^{-1}(V)$ is a μ-regular open set in X. This shows that f is a (μ, σ)-R-map. ■

Using Lemma 4.4, Lemma 4.1 and Theorem 4.1, as shown above, we conclude the following corollary.

Corollary 4.4. *Let f: $(X, \mu) \rightarrow (Y, \sigma)$ be a surjective, (μ, σ)-almost continuous and (μ, σ)-M-preopen function. If X is μ-nearly compact, then, Y is σ-nearly compact.*

CONCLUSION

We concluded that our objectives for this paper were achieved. Some decompositions of (μ, σ)-continuity on μ-nearly compact spaces are investigated. We also get some results by analysing the effect of mappings on μ-nearly compact spaces. The main result implies that the (μ, σ)-δ-continuous image of a μ-nearly compact space is σ-nearly compact.

CONFLICTS OF INTEREST

The authors declare no conflict of interest. The funders had no role in the design of the study; in the collection, analyses, or interpretation of data; in the writing of the manuscript, or in the decision to publish the results.

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